MATH 226: Differential Equations

TWO FUNDAMENTAL EXISTENCE AND UNIQUENESS THEOREMS

THEOREM 2.4.1(Linear First Order Differential Equations): If the functions p and g are continuous on an open interval I = (α, β) containing the point $t = t_0$, then there exists a unique function $y = \phi(t)$ that satisfies the differential equation

$$y' + p(t)y = g(t)$$

for each t in I, and that also satisfies the initial condition $y(t_0) = y_0$, where y_0 is an arbitrary prescribed initial value.

THEOREM 2.4.2(General First Oder Differential Equations): Let the functions f and $\partial f/\partial y$ be continuous in some rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ containing the point (t_0, y_0).

Then, in some interval $t_0 - h < t < t_0 + h$ contained in $\alpha < t < \beta$, there is a unique solution $y = \phi(t)$ of the initial value problem

$$y' = f(t,y), \quad y(t_0) = y_0.$$