**MATH 226: Notes on Assignment 3**

**Practice Problems 1.3:** 1-31 odd

**1**. The differential equation is second order, since the highest derivative in the equation is of order two. The equation is linear since the left hand side is a linear function of *y* and its derivatives and the right hand side is just a function of *t*.

**3**. The differential equation is fourth order since the highest derivative in the equation is of order four. The equation is linear since the left hand side is a linear function of *y* and its derivatives and the right hand side

**5**. The differential equation is second order since the highest derivative in the equation is of order two. The equation is nonlinear because of the term sin(*t* + *y*) which is not a linear function of *y*.

**7**. *a0 =1, a1 =1/(1+t), g* = 2 sin *t*; nonhomogeneous.

**9**. *a0 = x2, a1 = −3x, a2 = 4, g* = ln *x*; nonhomogeneous.

**11**. *a0 =1,a1 =0,a2* =cos *t*, *a3* =0, *a4* =1, *g* = e−*t*sin *t*; nonhomogenous.

**13**. If *y1* =e*t*, then y1′ =e*t* and y1′′ =e*t*. Therefore, y1′′−y1 =0. Also, y2 =cosh *t* implies y2′ =sinh t and y2′′ =cosh t. Therefore, y2′′ −y2 =0.

**15.** If *y = 3t + t2*, then *y′ =3+2t*. Thus, *ty′ −y = t (3 + 2t)−(3t + t2)= t2*.

**17**. If *y = t1/2*, then *y′ = t−1/2*/2 and *y′′ = −t−3/2*/4. Therefore,
 *2t2y′′ + 3ty′ − y = 2t2(−t−3/2/4)+3t(t−1/2/2)−t1/2 = (−1/2+3/2−1)t1/2* = 0.

Also, *y2 = t−1* implies *y2′ = −t−2* and *y′′ = 2t−3.* Therefore, *2t2y′′ + 3ty′ − y = 2t2(2t−3) + 3t(−t−2) − t−1 = (4 − 3 − 1)t−1* = 0.

**19**. If *y* = (cos *t*) ln cos *t* + *t* sin *t*, then *y′* = −(sin *t*) ln cos *t* + (cos *t*)(-sin *t*)/(cos *t*) + 1sin *t* + *t* cos t = −(sin *t*) ln cos *t –* sin *t* + sin *t* + *t* cos *t =*  −(sin *t*) ln cos *t* + *t* cos *t* and

*y′′* = −(cos *t* ) ln cos *t + (-*sin *t) (-*sin *t)/*cos *t*  +1 (cos *t* ) + *t(-*sin *t).*

Thus, y′′ + y = −(cos t) ln cos *t* − *t* sin *t* + sin2*t*/cos *t* + cos *t +*  (cos *t*) ln cos *t* + *t* sin *t*



**21**. Let *y* = e*rt*. Then *y′* = *r*e*rt*. Substituting these terms into the differential equation, we have *y′ +2y*=*r*e*rt* +2e*rt* =(*r*+2)e*rt* =0. This equation implies *r* = −2 since e*rt* is always positive.

**23.** Let *y* = e*rt*. Then *y′* = *r* e*rt* and *y′′* = *r*2 e*rt*. Substituting these terms into the differential equation, we have *y′′ + y′ − 6y* = (*r2 + r − 6*)*ert* = 0. In order for *r* to satisfy this equation, we need *r2 +r−6* = (*r* -2)(*r* +3) =0. That is, we need *r* = 2,−3.

**25.** Let *y = tr*. Then *y′ = rtr−1* and *y′′ = r(r − 1)tr−2*. Substituting these terms into the differential equation, we have *t2y′′ + 4ty′ + 2y = t2(r(r − 1)tr−2) + 4t(rtr−1) + 2tr = (r(r−1)+4r+2)tr* = 0. In order for *r* to satisfy this equation, we need *r(r−1)+4r+2* = 0. Simplifying this expression, we need *r2 + 3r + 2* = 0. The solutions of this equation are *r* = −1, −2.

**27.** If *y=Ce−2t*, then *y′ =−2Ce−2t*. Thus *y′ +2y*=0. Also,1=*y*(0)=*C*e0 =*C*.

**29**. If *y* = (sin *t*)/*t*2 + *C*/*t*2, then *y′* = (cos *t*)/*t*2 −2 (sin *t*)/*t*3 −2*C*/*t*3. Thus

*y′* +(2/*t*)*y* = (cos *t*)/*t*2. Also, 1/2 = *y*(1) = (sin 1)/1 + *C*/1 so *C* = ½ - sin 1.

