MATH 226 *Notes on Assignment 8*

Section 3.1: 13, 15, 23, 38

13. $\det(A - \lambda I) = \lambda^2 - \lambda - 2 = 0$, thus $\lambda = 2, -1$. First, $\lambda_1 = 2$ implies

$$A - \lambda_1 I = \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}.$$

Therefore,

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

is an eigenvector for λ_1. Second, $\lambda_2 = -1$ implies

$$A - \lambda_2 I = \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}.$$

Therefore,

$$\mathbf{x}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

is an eigenvector for λ_2.

15. $\det(A - \lambda I) = \lambda^2 - 2\lambda + 1 = 0$, thus $\lambda = 1$. Now, $\lambda = 1$ implies

$$A - \lambda_1 I = \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}.$$

Therefore,

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

is an eigenvector for λ.

23. $\det(A - \lambda I) = \lambda^2 + \lambda - 6 = 0$, thus $\lambda = 2, -3$. First, $\lambda_1 = 2$ implies

$$A - \lambda_1 I = \begin{pmatrix} -1 & 1 \\ 4 & -4 \end{pmatrix}.$$

Therefore,

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

is an eigenvector for λ_1. Second, $\lambda_2 = -3$ implies

$$A - \lambda_2 I = \begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix}.$$

Therefore,

$$\mathbf{x}_2 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

is an eigenvector for λ_2.

38. Suppose $\lambda = 0$ is an eigenvalue for A. Then $\lambda = 0$ is a solution of $\det(A - \lambda I) = 0$, i.e. $\det(A - 0 \cdot I) = \det(A) = 0$. Also, if $\det(A) = 0$, then $\det(A) = \det(A - 0 \cdot I) = 0$, and $\lambda = 0$ is an eigenvalue.

A.2: 1c, 7, 11, 12

1c: Row echelon form is the identity; rank is 3

7:

The augmented matrix of coefficients is $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & 1 & 2 & 1 \end{array}\right]$

and the reduced row echelon form is $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

but the last row asserts 0 = 1 so there is no solution.

11. We are looking for nontrivial values of c1, c2, c3 such that c1v1 + c2v2 + c3v3 = 0. Enter the vectors as columns in a matrix and reduce to row echelon form. The original matrix is $\begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ which reduces to $\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 5/2 \\ 0 & 0 & 0 \end{bmatrix}$. Thus we can let c1 = ½ c3, c2 = -5/2 and c3 can be arbitrary. Choosing c3 =2, gives c1 = 1, c2 = -5. Thus v1 – 5v2 + 2v3 = 0 so the set of vectors {v1, v2, v3} is linearly dependent.

12. The original matrix $\begin{bmatrix} 1 & -1 & -2 & -3 \\ 2 & 0 & -1 & 0 \\ 2 & 3 & 1 & -1 \\ 3 & 1 & 0 & 3 \end{bmatrix}$ reduces to $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ so $-2\mathbf{v}_1 + 3\mathbf{v}_2 - 4\mathbf{v}_3 + \mathbf{v}_4 = 0$ is one dependency relationship

A.3: 11. Det (A) = 20, Det(B) = -12 and AB = $\begin{bmatrix} -1 & 9 & 7 \\ 12 & 6 & 3 \\ 4 & -6 & -3 \end{bmatrix}$ has determinant -240.

A.4: 1, 4, 10

1. Characteristic polynomial is $\lambda^2 - 6\lambda + 8 = (\lambda - 2)(\lambda - 4)$ so eigenvalues are $\lambda = 2$ and $\lambda = 4$. An eigenvector for $\lambda = 2$ is $(1, 3)^T$ and an eigenvector for $\lambda = 4$ is $(1, 1)^T$.

4. Characteristic polynomial is $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = (\lambda - 1)(\lambda - 2)(\lambda - 3)$ so eigenvalues are $\lambda = 1$, $\lambda = 2$, and $\lambda = 3$
An eigenvector for $\lambda = 1$ is $(-1,0,1)^T$, an eigenvector for $\lambda = 2$ is $(-2, 1,0)^T$ and an eigenvector for $\lambda = 3$ is $(0,-1,1)^T$

10. Characteristic polynomial is $\lambda^3 - 2\lambda^2 - 2\lambda + 4 = (\lambda - 2)(\lambda^2 - 2)$ so eigenvalues are $\lambda = 2$, $\lambda = \sqrt{2}$, and $\lambda = -\sqrt{2}$.
An eigenvector for $\lambda = 2$ is $(0,1,0)^T$, an eigenvector for $\lambda = \sqrt{2}$, is $(\sqrt{2} + 1, 0, 1)^T$ and an eigenvector for $\lambda = -\sqrt{2}$, is $(-\sqrt{2+1}, 0, 1)^T$