

# Some Examples With Complex Eigenvalues

## Example from Class 18

```
syms x(t) y(t)
a = 2; b = 5/2; c = -1; d = 3;
ode1 = diff(x) == a*x + b*y
```

ode1(t) =

$$\frac{\partial}{\partial t} x(t) = 2x(t) + \frac{5y(t)}{2}$$

```
ode2 = diff(y) == c*x + d*y
```

ode2(t) =

$$\frac{\partial}{\partial t} y(t) = 3y(t) - x(t)$$

```
odes = [ode1; ode2]
```

odes(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t} x(t) = 2x(t) + \frac{5y(t)}{2} \\ \frac{\partial}{\partial t} y(t) = 3y(t) - x(t) \end{pmatrix}$$

```
S = dsolve(odes)
```

```
S = struct with fields:
  y: [1x1 sym]
  x: [1x1 sym]
```

```
xSol(t) = S.x
```

xSol(t) =

$$C_1 \left( \frac{\cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} + \frac{3 \sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} \right) + C_2 \left( \frac{3 \cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} - \frac{\sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} \right)$$

```
ySol(t) = S.y
```

ySol(t) =

$$C_1 \cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}} - C_2 \sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}$$

```
[xSol(t), ySol(t)] = dsolve(odes)
```

xSol(t) =

$$C_1 \left( \frac{\cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} + \frac{3 \sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} \right) + C_2 \left( \frac{3 \cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} - \frac{\sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{2} \right)$$

ySol(t) =

$$C_1 \cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}} - C_2 \sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}$$

```
cond1 = x(0) == 1
```

```
cond1 = x(0) = 1
```

```
cond2 = y(0) == 1
```

```
cond2 = y(0) = 1
```

```
conds = [cond1;cond2]
```

```
conds =
```

$$\begin{pmatrix} x(0) = 1 \\ y(0) = 1 \end{pmatrix}$$

```
[xSol(t), ySol(t)] = dsolve(odes,conds)
```

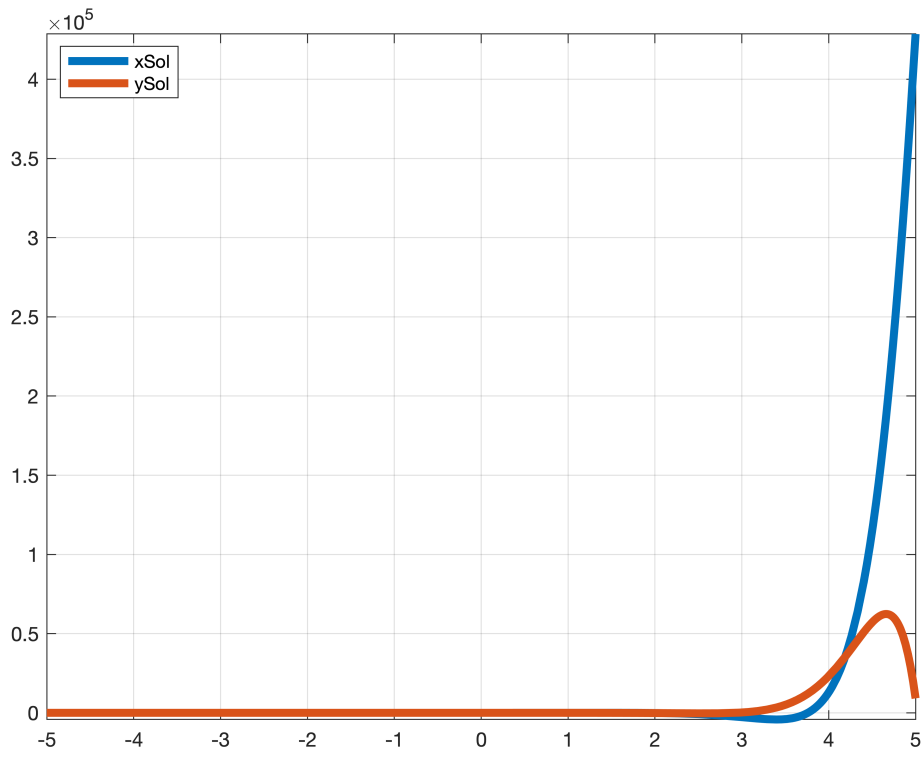
xSol(t) =

$$\cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}} + \frac{4 \sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{3}$$

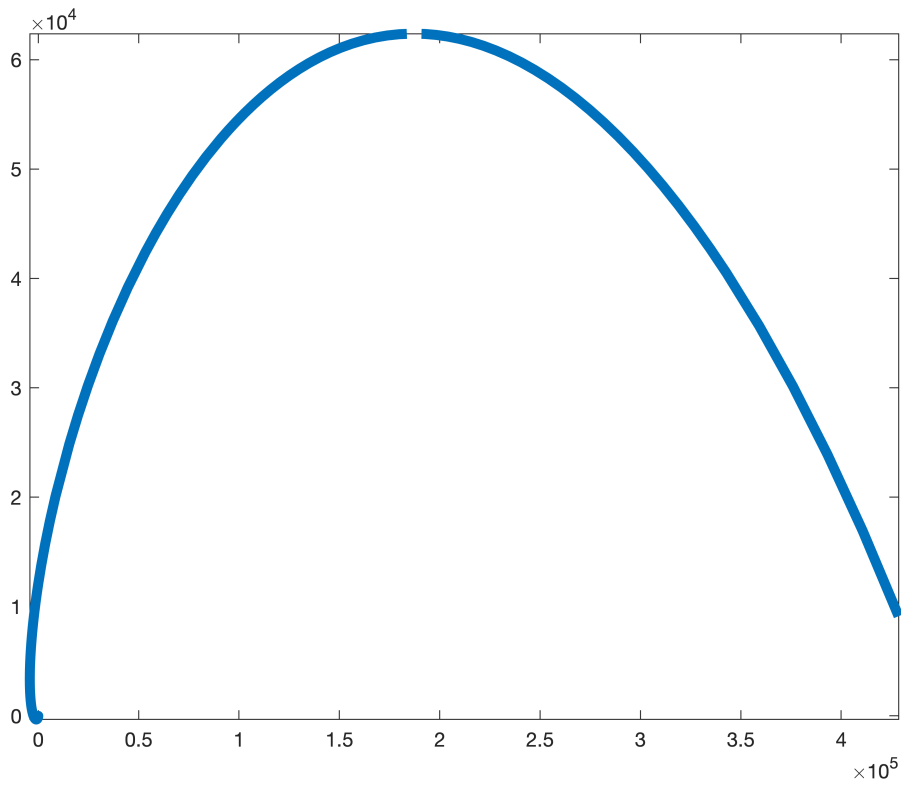
ySol(t) =

$$\cos\left(\frac{3t}{2}\right) e^{\frac{5t}{2}} - \frac{\sin\left(\frac{3t}{2}\right) e^{\frac{5t}{2}}}{3}$$

```
fplot(xSol, 'LineWidth', 4)
hold on
fplot(ySol, 'LineWidth', 4)
grid on
legend('xSol','ySol', 'Location','best')
hold off
```



```
fplot(xSol, ySol, 'LineWidth', 5)
```



## Exercise 8 in Section 3.4

```
a = 2; b = -5; c = 1 ; d = -2;  
ode1 = diff(x) == a*x + b*y
```

```
ode1(t) =
```

$$\frac{\partial}{\partial t} x(t) = 2x(t) - 5y(t)$$

```
ode2 = diff(y) == c*x + d*y
```

```
ode2(t) =
```

$$\frac{\partial}{\partial t} y(t) = x(t) - 2y(t)$$

```
odes = [ode1; ode2]
```

```
odes(t) =
```

$$\begin{pmatrix} \frac{\partial}{\partial t} x(t) = 2x(t) - 5y(t) \\ \frac{\partial}{\partial t} y(t) = x(t) - 2y(t) \end{pmatrix}$$

```
S = dsolve(odes)
```

```
S = struct with fields:  
  y: [1x1 sym]  
  x: [1x1 sym]
```

```
xSol(t) = S.x
```

$$xSol(t) = C_2 (2 \cos(t) - \sin(t)) - C_1 (\cos(t) + 2 \sin(t))$$

```
ySol(t) = S.y
```

$$ySol(t) = C_2 \cos(t) - C_1 \sin(t)$$

```
[xSol(t), ySol(t)] = dsolve(odes)
```

$$xSol(t) = C_2 (2 \cos(t) - \sin(t)) - C_1 (\cos(t) + 2 \sin(t))$$

$$ySol(t) = C_2 \cos(t) - C_1 \sin(t)$$

```
cond1 = x(0) == 1
```

$$cond1 = x(0) = 1$$

```
cond2 = y(0) == 1
```

$$cond2 = y(0) = 1$$

```
conds = [cond1; cond2]
```

```
conds =
```

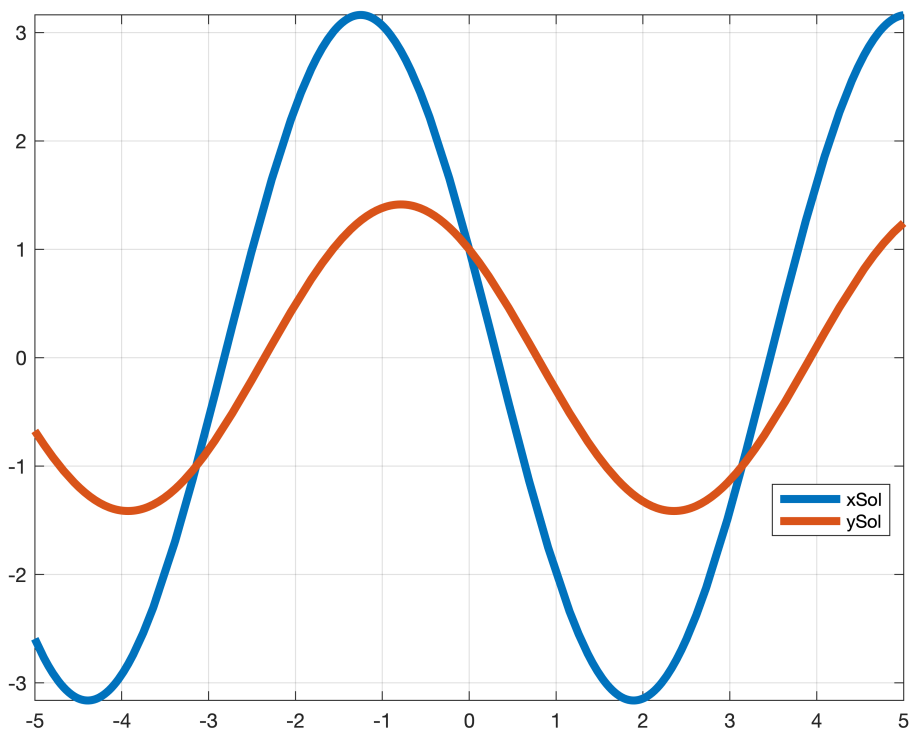
$$\begin{cases} x(0) = 1 \\ y(0) = 1 \end{cases}$$

```
[xSol(t), ySol(t)] = dsolve(odes,conds)
```

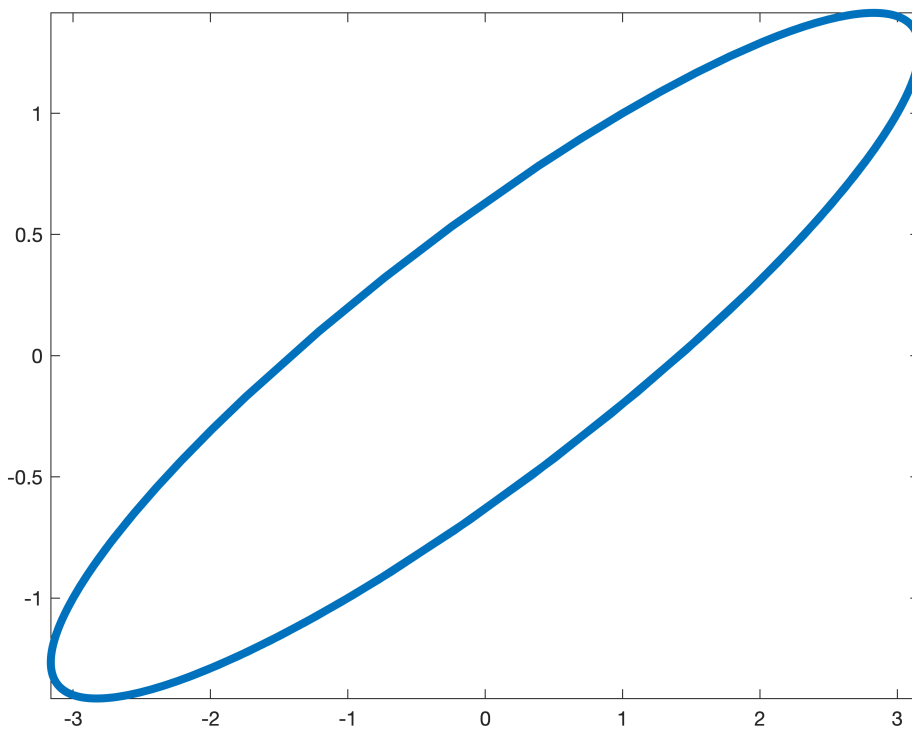
$$xSol(t) = \cos(t) - 3 \sin(t)$$

$$ySol(t) = \cos(t) - \sin(t)$$

```
fplot(xSol, 'LineWidth', 4)
hold on
fplot(ySol, 'LineWidth', 4)
grid on
legend('xSol','ySol', 'Location','best')
hold off
```



```
fplot(xSol, ySol, 'LineWidth', 4)
```



### Exercise 5 from Section 3.4

```
a = 1; b = -1; c = 5 ; d = -3;
ode1 = diff(x) == a*x + b*y
```

```
ode1(t) =
```

$$\frac{\partial}{\partial t} x(t) = x(t) - y(t)$$

```
ode2 = diff(y) == c*x + d*y
```

```
ode2(t) =
```

$$\frac{\partial}{\partial t} y(t) = 5 x(t) - 3 y(t)$$

```
odes = [ode1; ode2]
```

```
odes(t) =
```

$$\begin{pmatrix} \frac{\partial}{\partial t} x(t) = x(t) - y(t) \\ \frac{\partial}{\partial t} y(t) = 5 x(t) - 3 y(t) \end{pmatrix}$$

```
S = dsolve(odes)
```

*S = struct with fields:*

```
y: [1×1 sym]
x: [1×1 sym]
```

```
xSol(t) = S.x
```

```
xSol(t) =
```

$$C_1 \left( \frac{2e^{-t} \cos(t)}{5} - \frac{e^{-t} \sin(t)}{5} \right) - C_2 \left( \frac{e^{-t} \cos(t)}{5} + \frac{2e^{-t} \sin(t)}{5} \right)$$

```
ySol(t) = S.y
```

$$ySol(t) = C_1 e^{-t} \cos(t) - C_2 e^{-t} \sin(t)$$

```
[xSol(t), ySol(t)] = dsolve(odes)
```

```
xSol(t) =
```

$$C_1 \left( \frac{2e^{-t} \cos(t)}{5} - \frac{e^{-t} \sin(t)}{5} \right) - C_2 \left( \frac{e^{-t} \cos(t)}{5} + \frac{2e^{-t} \sin(t)}{5} \right)$$

$$ySol(t) = C_1 e^{-t} \cos(t) - C_2 e^{-t} \sin(t)$$

```
cond1 = x(0) == 1
```

$$\text{cond1} = x(0) = 1$$

```
cond2 = y(0) == 1
```

$$\text{cond2} = y(0) = 1$$

```
conds = [cond1;cond2]
```

```
conds =
```

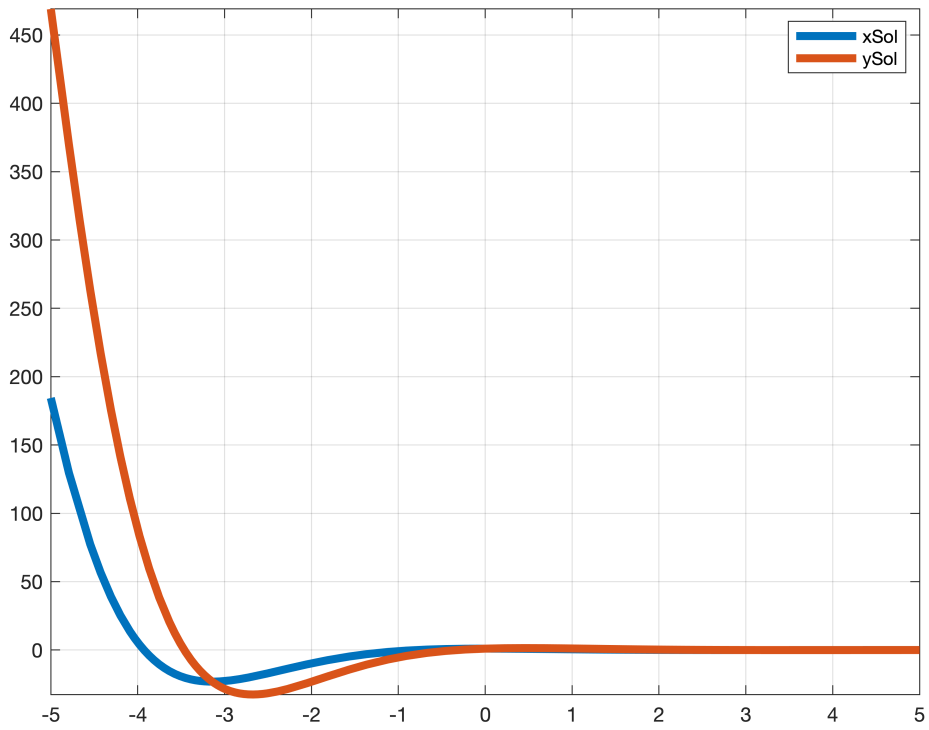
$$\begin{pmatrix} x(0) = 1 \\ y(0) = 1 \end{pmatrix}$$

```
[xSol(t), ySol(t)] = dsolve(odes,conds)
```

$$xSol(t) = e^{-t} \cos(t) + e^{-t} \sin(t)$$

$$ySol(t) = e^{-t} \cos(t) + 3e^{-t} \sin(t)$$

```
fplot(xSol, 'LineWidth', 4)
hold on
fplot(ySol, 'LineWidth', 4)
grid on
legend('xSol','ySol', 'Location','best')
hold off
```



```
fplot(xSol, ySol, 'LineWidth', 5)  
hold off
```

