

Examples of Repeated Eigenvalues

Example 2 of Section 3.5

```
A = [-1/2 1; 0 -1/2 ]
```

```
A = 2x2  
  -0.5000    1.0000  
     0    -0.5000
```

```
RHS = [0;0];  
B = rref([A -RHS]);  
eqSoln = B(:,end)
```

```
eqSoln = 2x1  
  0  
  0
```

```
[X D] = eig(sym(A))
```

```
X =
```

```
 $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ 
```

```
D =
```

```
 $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ 
```

```
yBounds = [-5,5]
```

```
yBounds = 1x2  
  -5     5
```

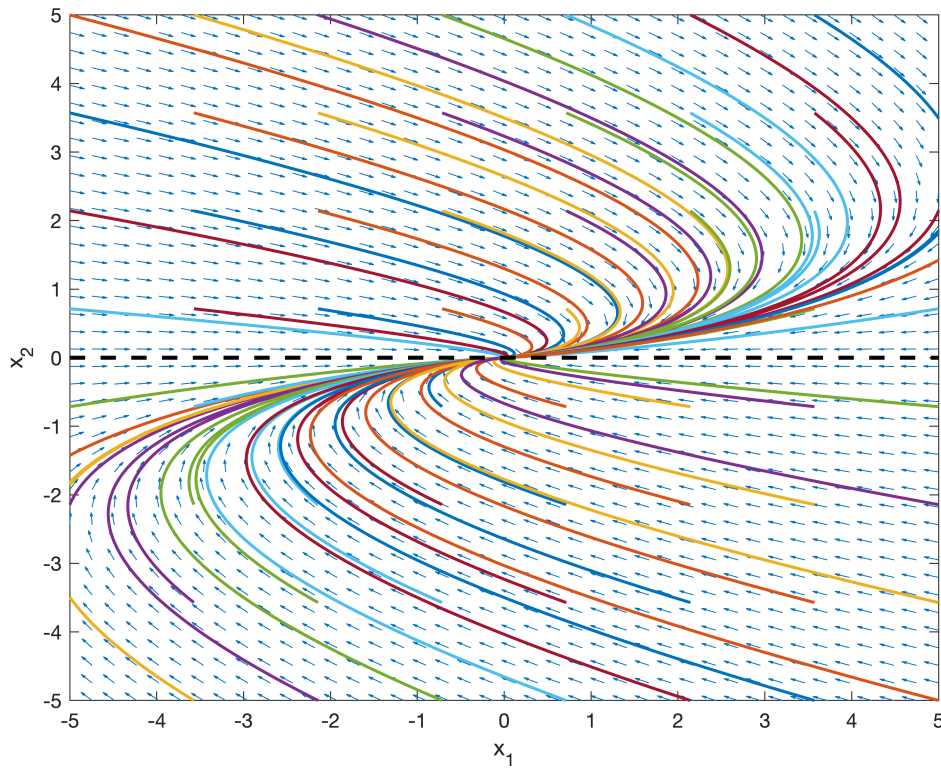
```
tBounds = [0,50]
```

```
tBounds = 1x2  
  0    50
```

```
M = 8
```

```
M = 8
```

```
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)  
hold off
```



```
syms x(t) y(t)
ode1 = diff(x) == -1/2*x + 1*y;
ode2 = diff(y) == -1/2 * y;
odes = [ode1; ode2]
```

```
odes(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t} x(t) = y(t) - \frac{x(t)}{2} \\ \frac{\partial}{\partial t} y(t) = -\frac{y(t)}{2} \end{pmatrix}$$

```

```
[xSol, ySol] = dsolve(odes)
```

```
xSol =
```

$$C_1 e^{-\frac{t}{2}} + C_2 t e^{-\frac{t}{2}}$$

```
ySol =
```

$$C_2 e^{-\frac{t}{2}}$$

```
cond1 = x(0) == 1;
cond2 = y(0) == 2;
conds = [cond1; cond2]
```

```
conds =
```

$$\begin{pmatrix} x(0) = 1 \\ y(0) = 2 \end{pmatrix}$$

```
[xSol, ySol]= dsolve(odes, conds)
```

```
xSol =
```

$$e^{-\frac{t}{2}} + 2te^{-\frac{t}{2}}$$

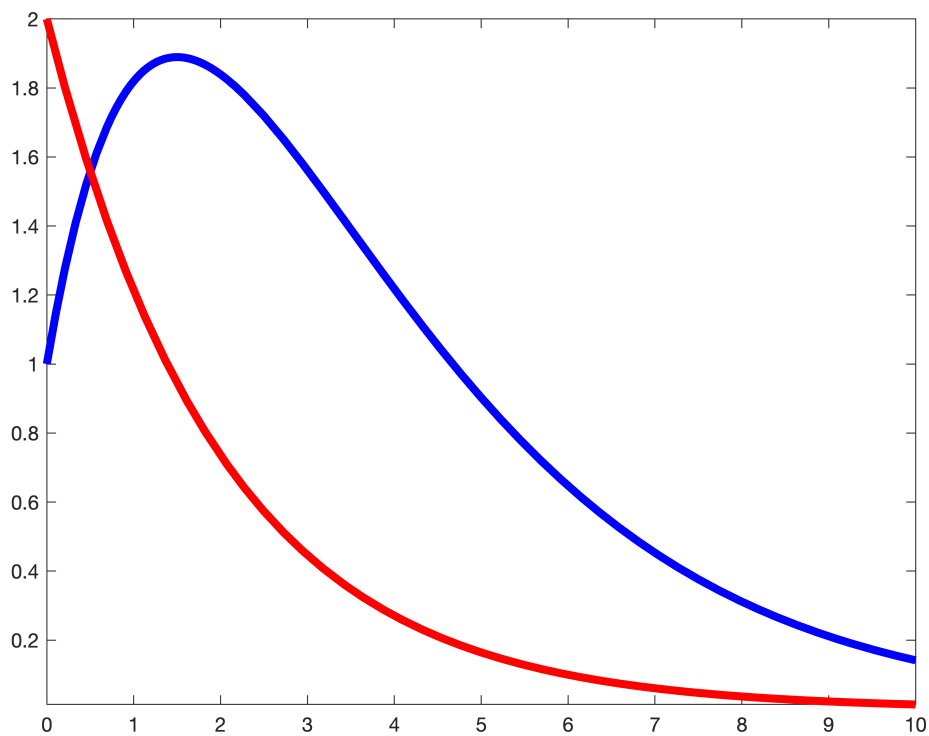
```
ySol =
```

$$2e^{-\frac{t}{2}}$$

```
fplot(xSol, [0,10], 'b', 'LineWidth', 4)
```

```
hold on
```

```
fplot(ySol, [0,10], 'r', 'LineWidth', 4)
```



Exercise 3 of Section 3.5

$$A = \begin{bmatrix} -3/2 & 1 \\ -1/4 & -1/2 \end{bmatrix}$$

$$A = \begin{matrix} 2 \times 2 \\ \begin{bmatrix} -1.5000 & 1.0000 \\ -0.2500 & -0.5000 \end{bmatrix} \end{matrix}$$

$$RHS = [0;0];$$

```
B = rref([A -RHS]);  
eqSoln = B(:,end)
```

```
eqSoln = 2x1  
 0  
 0
```

```
[X D] = eig(sym(A))
```

```
X =
```

```
(2  
 1)
```

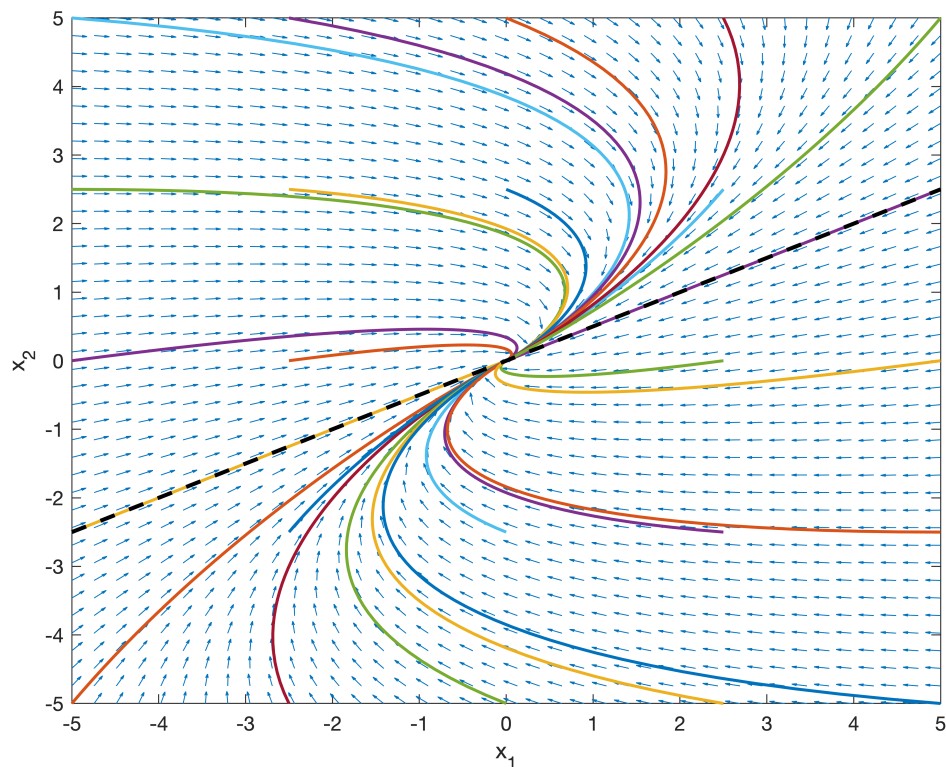
```
D =
```

```
(-1  0  
 0 -1)
```

```
M=5
```

```
M = 5
```

```
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)  
hold off
```



```
ode1 = diff(x) == -3/2*x + 1*y;  
ode2 = diff(y) == -1/4*x -1/2 * y;  
odes = [ode1; ode2]
```

```
odes(t) =
```

$$\begin{cases} \frac{\partial}{\partial t} x(t) = y(t) - \frac{3x(t)}{2} \\ \frac{\partial}{\partial t} y(t) = -\frac{x(t)}{4} - \frac{y(t)}{2} \end{cases}$$

```
[xSol, ySol] = dsolve(odes)
```

```
xSol =
```

$$C_2 \left(e^{-t} - \frac{t e^{-t}}{2} \right) - \frac{C_1 e^{-t}}{2}$$

```
ySol =
```

$$-\frac{C_1 e^{-t}}{4} - \frac{C_2 t e^{-t}}{4}$$

```
cond1 = x(0) == 1;
```

```
cond2 = y(0) == 1;
```

```
conds = [cond1; cond2]
```

```
conds =
```

$$\begin{pmatrix} x(0) = 1 \\ y(0) = 1 \end{pmatrix}$$

```
[xSol, ySol]= dsolve(odes, conds)
```

```
xSol =
```

$$e^{-t} + \frac{t e^{-t}}{2}$$

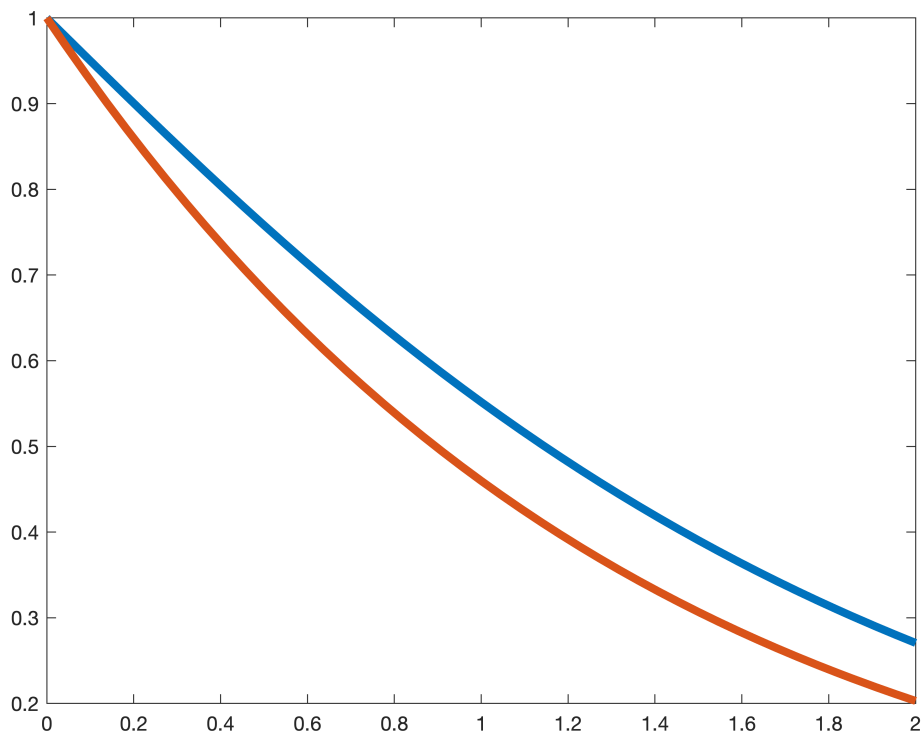
```
ySol =
```

$$e^{-t} + \frac{t e^{-t}}{4}$$

```
fplot(xSol, [0,2], 'LineWidth', 4)
```

```
hold on
```

```
fplot(ySol, [0,2], 'LineWidth', 4)
```



Example from Class 19 (October 26)

```
A = [4 1/2; -2 6]
```

```
A = 2x2
 4.0000    0.5000
-2.0000    6.0000
```

```
RHS = [0;0];
B = rref([A -RHS]);
eqSoln = B(:,end)
```

```
eqSoln = 2x1
 0
 0
```

```
[X D] = eig(sym(A))
```

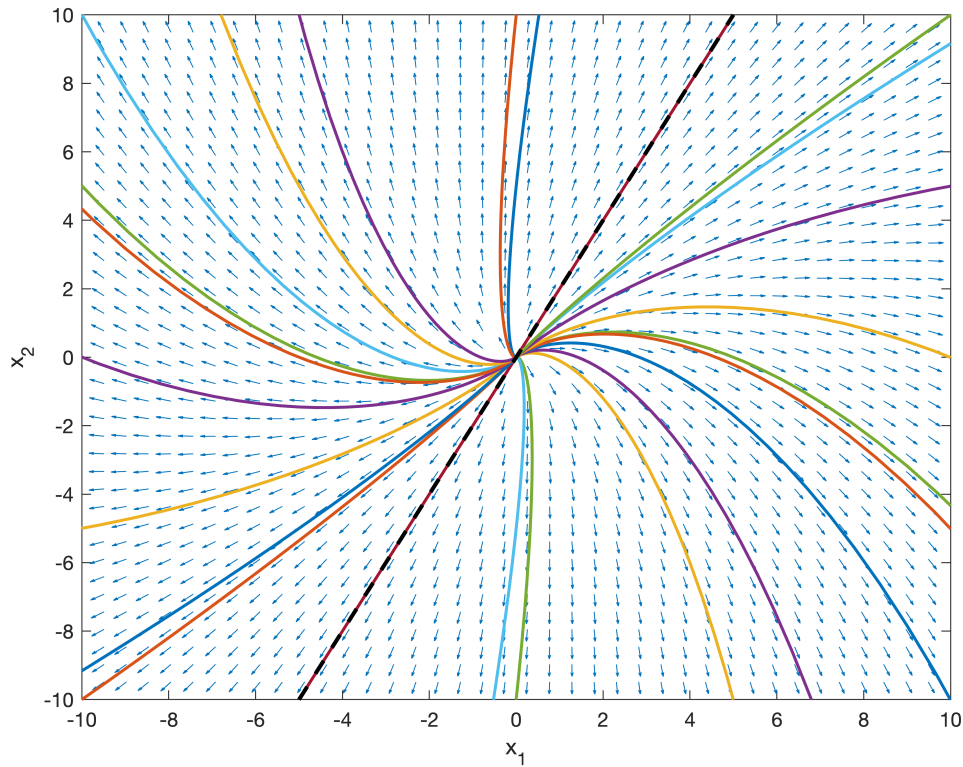
```
X =
( 1/2 )
( 1 )
D =
```

$$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

```
yBounds = [-10,10];
tBounds = [-10,10];
M=5
```

M = 5

```
plotPhasePlane(A, yBounds, tBounds,M,RHS,eqSoln)
hold off
```



```
ode1 = diff(x) == 4*x + (1/2)*y;
ode2 = diff(y) == -2*x + 6*y;
odes = [ode1; ode2]
```

odes(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t} x(t) = 4x(t) + \frac{y(t)}{2} \\ \frac{\partial}{\partial t} y(t) = 6y(t) - 2x(t) \end{pmatrix}$$

```
[xSol, ySol] = dsolve(odes)
```

$$xSol = C_2 (e^{5t} - t e^{5t}) - C_1 e^{5t}$$

$$ySol = -2 C_1 e^{5t} - 2 C_2 t e^{5t}$$

```
cond1 = x(0) == 4;  
cond2 = y(0) == 6;  
conds = [cond1; cond2]
```

```
conds =  
(  
  x(0) = 4  
  y(0) = 6  
)
```

```
[xSol, ySol]= dsolve(odes, conds)
```

```
xSol = 4 e5t - t e5t  
ySol = 6 e5t - 2 t e5t
```

```
fplot(xSol, [0,1], 'LineWidth', 4)  
hold on  
fplot(ySol, [0,1], 'LineWidth', 4)
```

