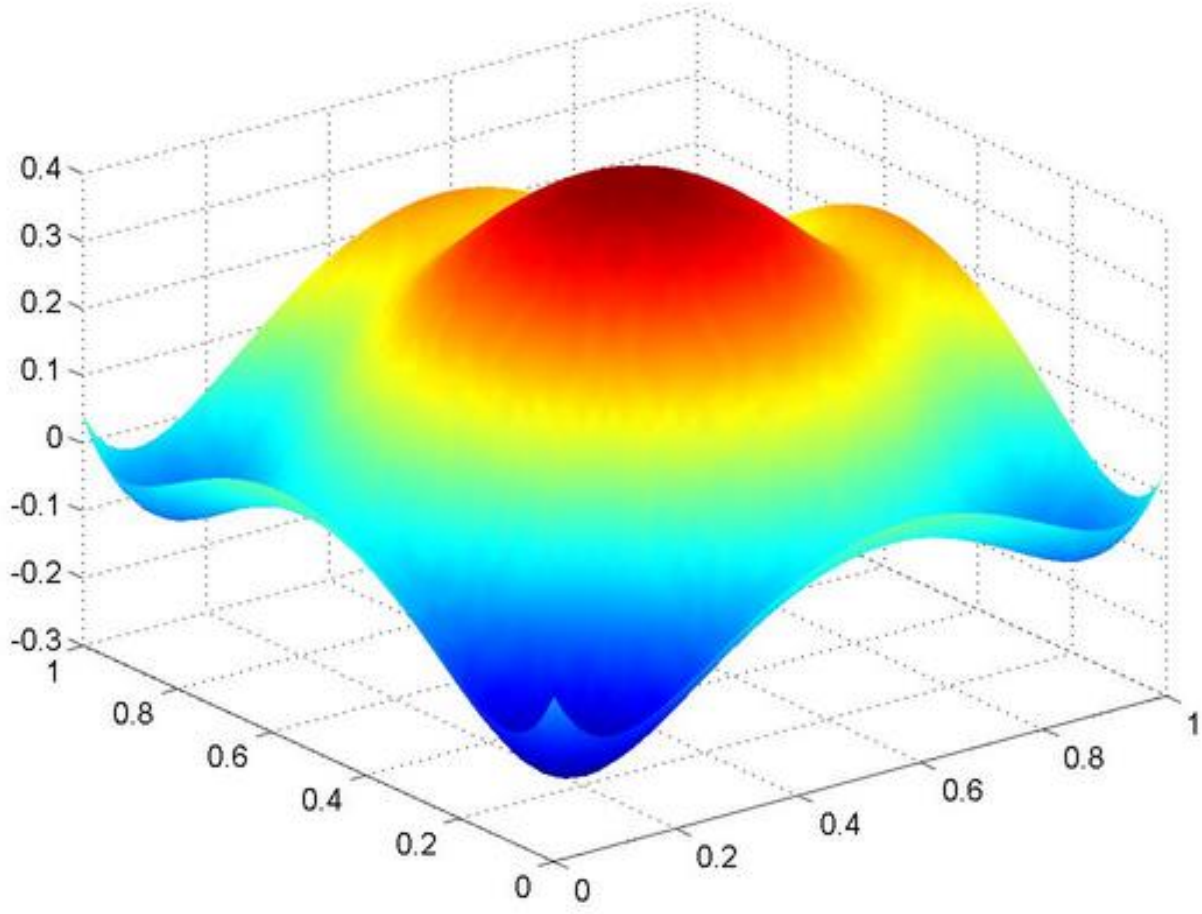
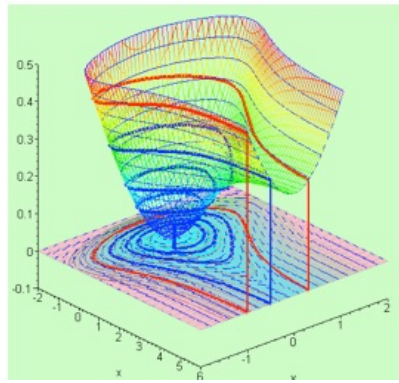


It is a curious historical fact that modern quantum mechanics began with two quite different mathematical formulations: the **differential equations** of Schroedinger and the **matrix algebra** of Heisenberg. The two apparently dissimilar approaches were proved to be mathematically equivalent. *Richard Feynman*

MATH 226: Differential Equations

Michael Olinick
Middlebury College



September 12, 2022

What Are Differential Equations?

A **differential equation** is an equation relating some unknown function and one or more of its derivatives.

- **Orindary** differential equation (ODE): unkown function has *only* one independent variable.

$$y = y(t), \frac{dy}{dt} = ky$$

- **Partial** differential equation (PDE): unknown function has more than one independent variable.

$$u = u(x, y), u_{xx} + u_{yy} = 0 (or \Delta u = 0)$$

What Are Differential Equations?

The **order** of a differential equation is the order of the highest derivative appearing in the equation.

All differential equations can be written in the form

$$F(\text{independent variable, dependent variable, variable and derivatives}) = 0$$

where all derivatives up to the highest power in the equation are variables in F.

$$\frac{dy}{dt} = ky, \frac{dy}{dt} - ky = 0, F(t, y, \frac{dy}{dt}) = \frac{dy}{dt} - ky$$

$$u_{xx} + u_{yy} = 0, F(x, y, u_x, u_y, u_{xx}, u_{yy}) = u_{xx} + u_{yy}$$

What is a Solution To a Differential Equation?

Give the ODE

$$F(t, y, y', y'', ..., y^{(n)}) = 0$$

a **solution** is a function $y = \phi(t)$ satisfying the equation for all t in some open interval I:

1. ϕ is n times differentiable in I.
2. ϕ satisfies the equation for all t in I.

We say that $y = \phi(t)$ is **a solution to the differential equation** on I.

A Differential Equation and Solution

$$\textit{Equation} : \frac{dy}{dt} = 12y$$

$$\textit{Solution} : \phi(t) = 9e^{12t}$$

$$\textit{Check} : \phi'(t) = 9(12e^{12t}) = 12(9e^{12t}) = 12\phi(t)$$

Why Do We Care About Differential Equations?

Among all of the mathematical disciplines the theory of differential equations is the most important... It furnishes the explanation of all those elementary manifestations of nature which involve time.

Sophus Lie

Why Do We Care About Differential Equations?

Among all of the mathematical disciplines the theory of differential equations is the most important... It furnishes the explanation of all those elementary manifestations of nature which involve time.

Sophus Lie

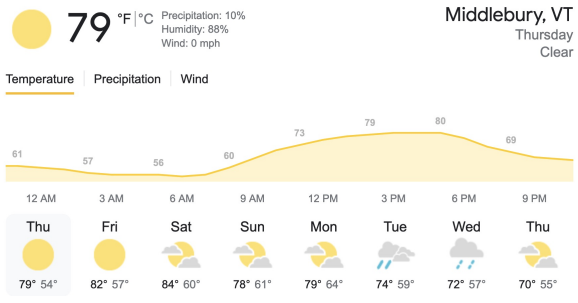
Sophus Lie

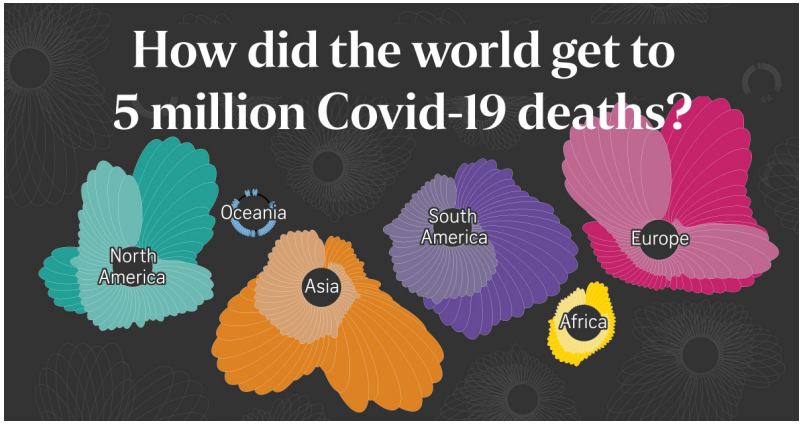
Born:December 17, 1842, Nordfjordeid, Norway

Died: February 18, 1899, Oslo, Norway

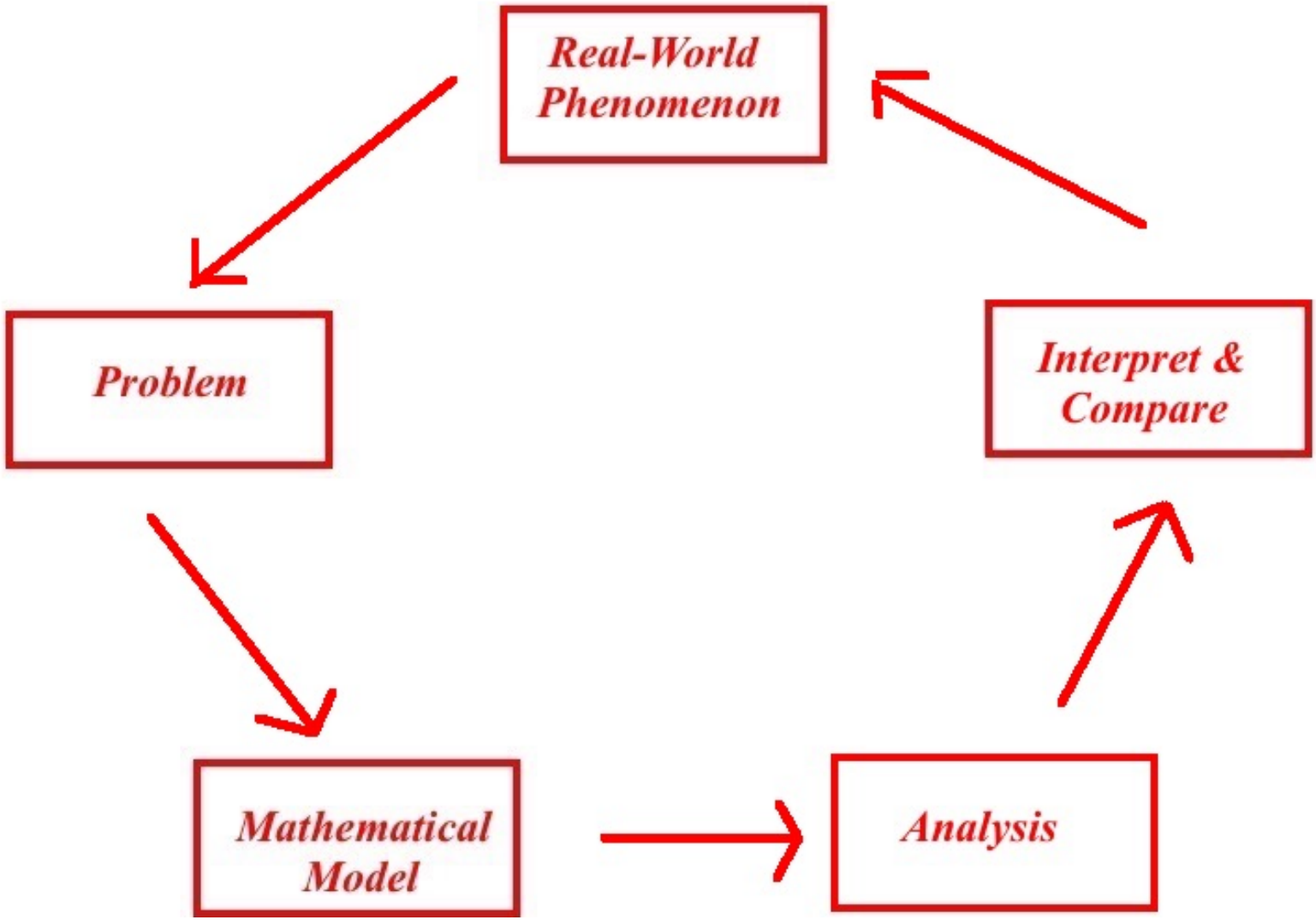
MacTutor Biography

Why Do We Care About Differential Equations?

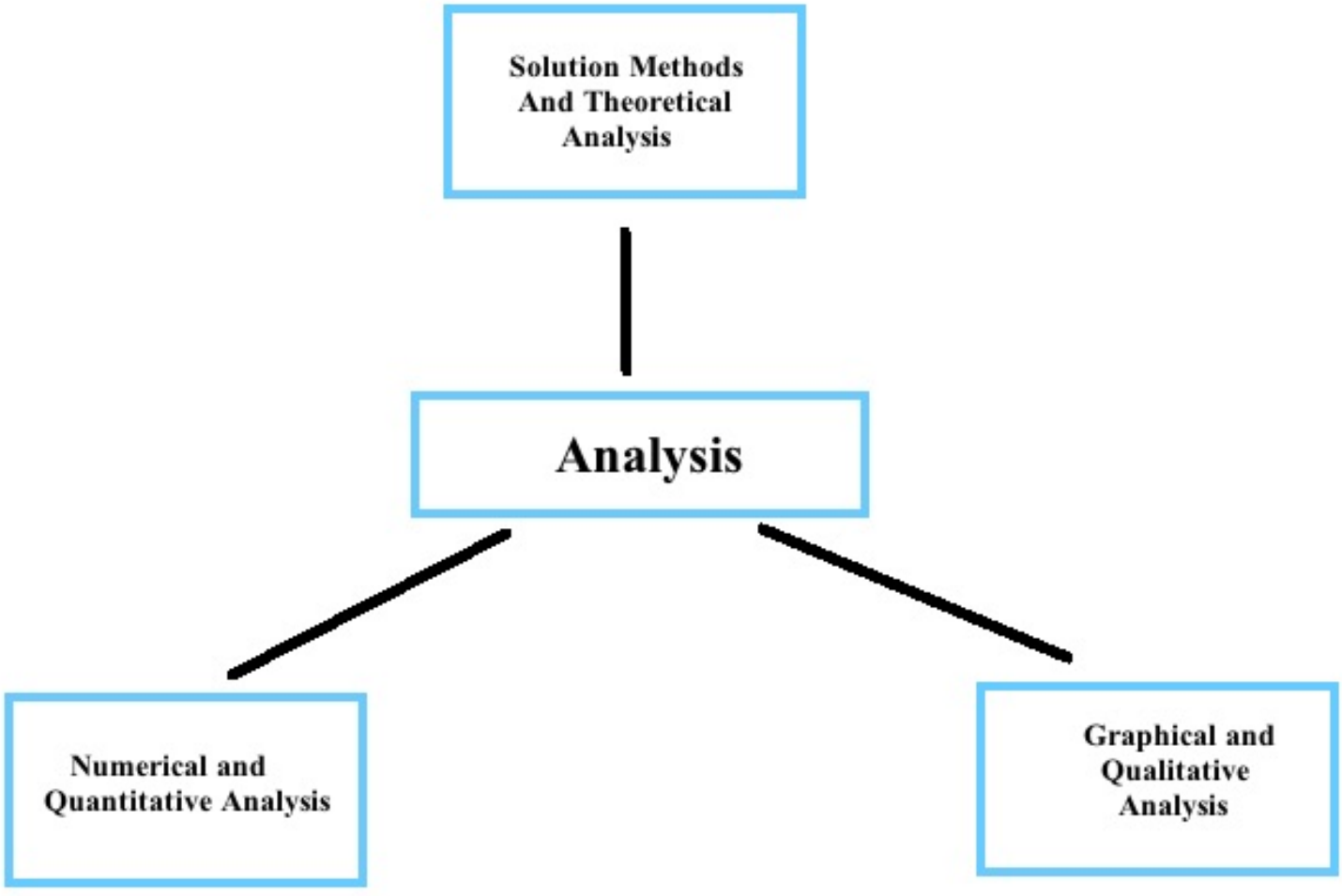




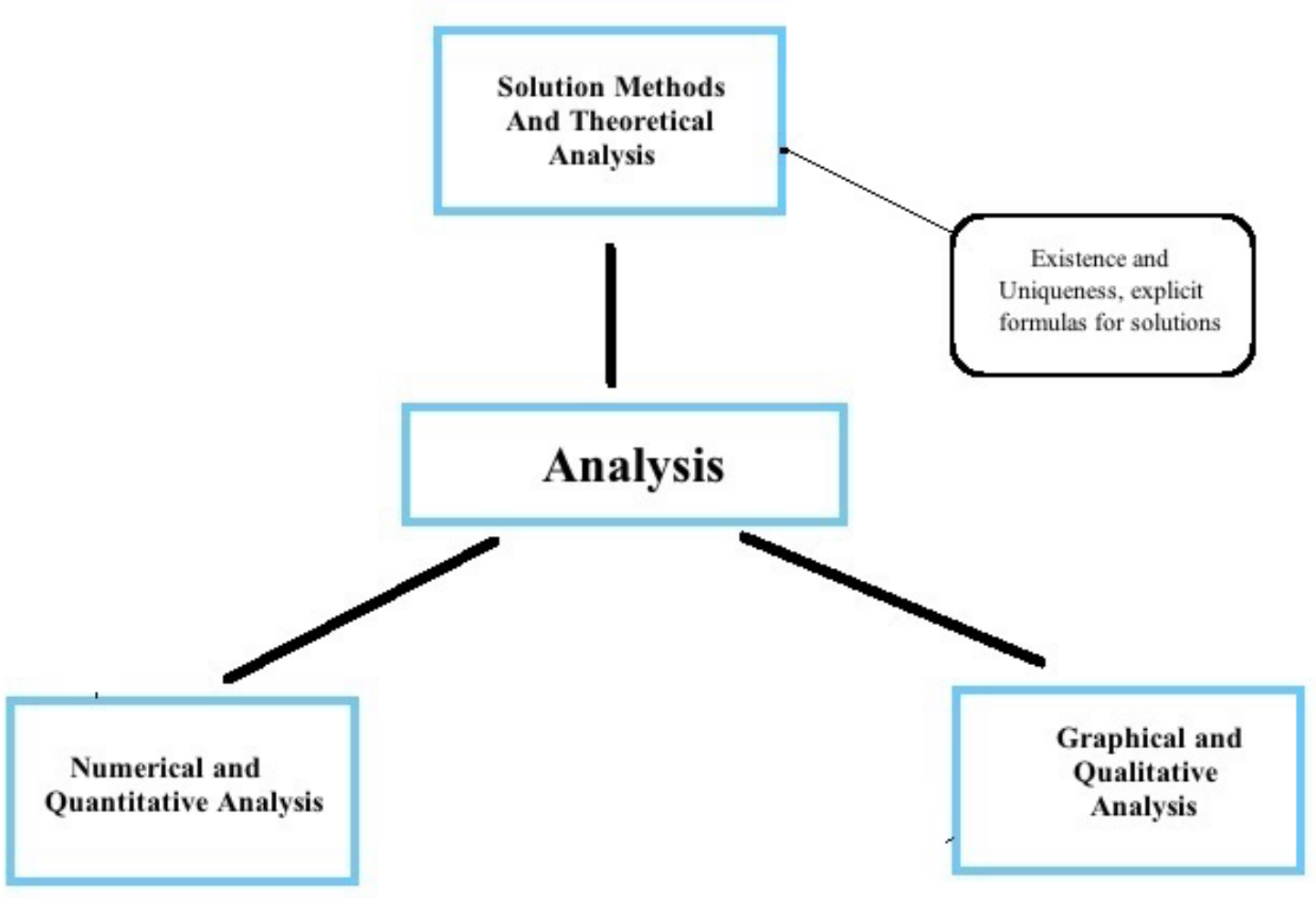
Mathematical Modeling



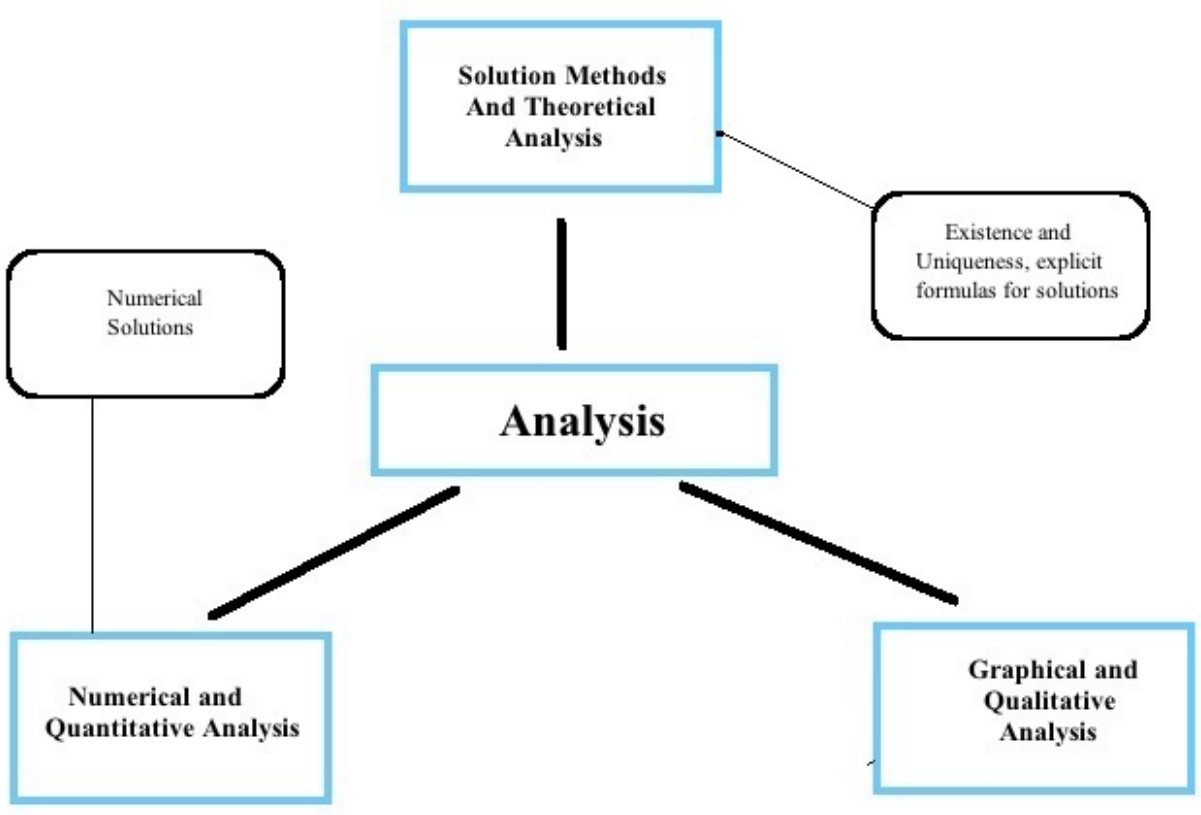
Analyzing Differential Equations



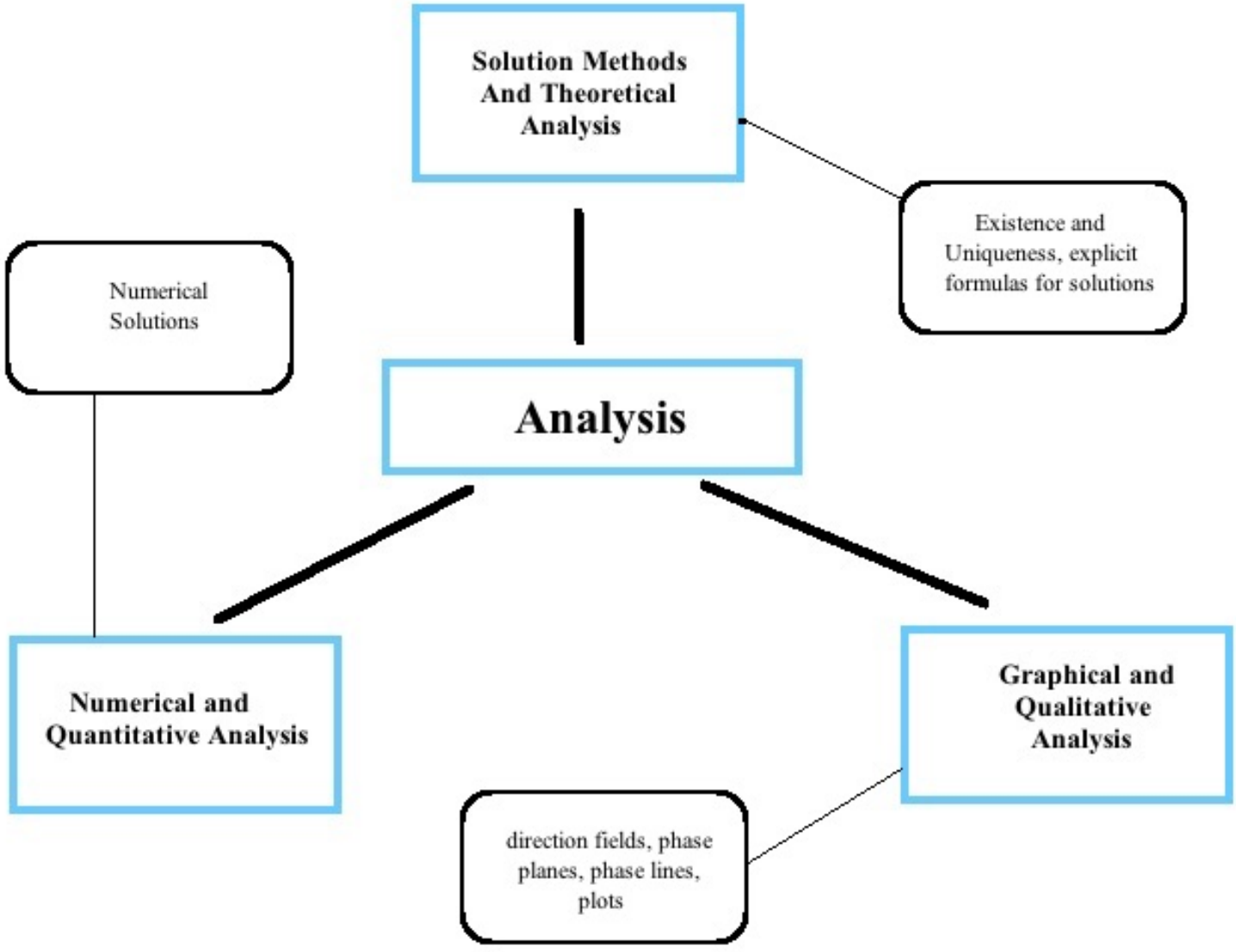
Analyzing Differential Equations



Analyzing Differential Equations



Analyzing Differential Equations



What is a Differential Equation(Informally)?

An equation that gives some explicit information about the **derivative** of a function. but not about the function itself.

Goal: Solve the equation to find the underlying function.

Example 1

$$y' = 2x, \frac{dy}{dx} = 2x, f'(x) = 2x$$

What are the possibilities for f?

$f(x) = x^2 + C$ where C is any constant

Note: We can always check our proposed answer.
Can there be any other solution?

Example 2: Generalize Example 1

$$y' = g(x), \frac{dy}{dx} = g(x), f'(x) = g(x)$$

Solution:

$$y = f(x) = \int g(x)dx$$

The Integration (or Antiderivative) Problem

Techniques:

Substitution = Change of Variable

Integration By Parts

Partial Fraction Decomposition

Where Do Differential Equations Arise?
Derivative is Measure of Rate of Change

Physical laws may give us information on how things evolve over time.
Derivatives will be with respect to time.
Notation:
Independent Variable: t, x
Dependent Variable: y, P, u

Example 3

$$P'(t) = 3P(t) \text{ with } P(0) = 100$$

Initial Value Problem

Applications:

Colony of Bacteria

Money Compounded Continuously

Human Population with Constant Per Capita Growth Rate

We'll Start Here Next Time