MATH 223 Fall 2022

Independent Project 1
Complete by Friday, October 7

Would Dwayne "The Rock" Make the Jump or Die?

To answer this question and find the Rock's trajectory, we need to compute a parabola that connects three coordinates on the *Skyscraper* poster: Johnson's destination, his current position and his starting point.

First, we need a unit of distance. Johnson is 6-foot-5, so let's give him a 7-foot span from his feet up to his raised hands. We will call this unit 1 Rock.

Next, we need to identify where he's aiming. He's clearly not trying to land his feet in the window—that would just be cinematically lazy—so let's say his destination is 1 Rock below the bottom of the window, so he can *just barely* grab hold of the edge, shattered glass be damned. Let's set our axes so that point is at the origin (0,0).

Using 1 Rock as the measurement unit, the poster (see below) indicates that his starting point (the end of the crane) is (5,4) and his mid-air position is (3,3). [You may do your own measurements and come up with slightly different estimates; feel free to work out the problem with your figures as the coordinates.]

Questions To Investigate

1. From our analysis in class, we know that The Rock's path follows a parabola with an equation of the form

$$y = ax^2 + bx + c$$

for some constants *a*, *b c*. Using assumptions that (0,0), (3,3) and (5,4) are on the parabola, determine the values of *a, b,* and *c.*

2. Sketch a graph of the parabola you have determined.

3. The parabola does not completely solve our problem. We need to deduce the Rock's necessary initial velocity. How fast must he run to make the jump so that he actually follows the parabola. Is that initial speed actually within the realm of human possibility? You will derive the horizontal and vertical parametric equations of motion. Let *h* be the initial horizontal velocity (what we denoted as | **w** | cos θ in class). Explain why *h* is negative in this case. Let *t* represent the time in seconds after the jump begins. Discuss why the horizontal position is given by

$$x(t) = ht + 5$$

where distance is measured in Rocks.

4. If *v* is the initial vertical velocity (| **w** | sin θ), show that the vertical position is given by

$$y(t) = -\frac{g}{2}t^2 + vt + 4$$

where g is the gravitational constant. In our case, the units on g should rocks/sec/sec. Explain why our value of g is 4.6.

5. Show that we can also write the equation for the parabola of motion as

$$y = \frac{-2.3}{h^2}(x-5)^2 + \frac{v}{h}(x-5).$$

6. By equating coefficients, or otherwise, determine that *h* is about $-\sqrt{23} = -4.8$ Rocks per second. Thus The Rock would be running at a speed of 4.8 Rocks per second when he jumps off the crane.

7. Convert 4.8 Rocks per second into miles per hour or kilometers per hour. Can humans run this fast?
[The current world record for the 100 **meter** dash is 9.58 seconds, set by Usain Bolt in 2009. Asafa Powell set the world record of 9.07 seconds for the 100 **yard** dash in 2010.]

8. With $x(t) = -4.8\,t + 5$ and $y(t) = -2.3t^2 + vt + 4$, determine how long The Rock was in the air before he hit the bottom of the window. How fast was he going when he hit the hall?

9. Using the equations from Question 8, show that *v* is negative and determine its value.

10. If the data on The Rock's positions are correct, then discuss why your analysis shows that he actually jumped **downward** off the crane at about 6.9 mph so his vertical position is always decreasing.

11. But…. The video (about the 7:20 point in https://www.youtube.com/watch?v=Rnam_wHwVWE) shows The Rock **leaping upward** as he leaves the crane so he stays above the height of the crane for some time before he begins to descend.

Thus the estimate of (3,3) as a mid-air position can not be correct. Keeping (5,4) and (0,0) as the initial and final positions, respectively, it has been claimed that "He could also make it if he jumps upward or directly forward with a slightly slower initial velocity." Determine at least one initial vector (**w**) that would enable The Rock to be successful with $|h|$ less than 4.8 Rocks/sec. Find an equation for the corresponding parabola of motion and sketch its graph. Compute the time The Rock is in the air and his impact velocity. Could he withstand such an impact without injury?

12. Our model of projectile motion ignores all forces except for gravity. Discuss how you might also include *air resistance* as a force into the equations for horizontal and vertical acceleration. How would your representation of such a force affect the equations for $x(t)$ and $y(t)$. Do you think the path of motion would still be a parabola?