MATH 223: Multivariable Calculus

Class 8

September 28, 2022

- Notes on Assignment 7
- Assignment 8
- Unified Treatment Of Tangent Lines and Planes
- Parametrized Surfaces in *Maple* [Handouts Folder]

Announcements

Exam 1: Next Monday, 7 PM - No Time Limit

No Books, Notes, Computers, etc.

Where to Find the Maple Files

Classes	Sep 1, 2022 at 9:53 AM
Fall20	Mar 9, 2021 at 10:12 PM
Fall21	Dec 22, 2021 at 12:34 PM
Fall22	Sep 22, 2022 at 2:33 PM
MATH0223A	Sep 19, 2022 at 10:34 AM
DROPBOX	Mar 21, 2022 at 4:29 PM
HANDOUTS	Today at 12:12 AM
Circles and Ellipses.mw	Feb 15, 2022 at 12:45 PM
Class 3.mw	Feb 19, 2022 at 11:48 AM
Class 5 Examples.maple	Sep 21, 2022 at 11:59 PM
Class 5.pdf	Sep 20, 2022 at 1:30 PM
LimitExample.maple	Sep 22, 2022 at 6:26 PM
Parametrized Surfaces.mw	Feb 27, 2022 at 1:52 PM
MATH SPR20 Archive	Sep 19, 2022 at 10:48 AM
PUBLIC_HTML	Sep 21, 2022 at 9:16 AM
RETURN	Sep 12, 2022 at 2:29 PM
SHARE	Mar 21, 2022 at 4:29 PM
WORKSPACE	Sep 12, 2022 at 2:29 PM
MATH0226A	Mar 21, 2022 at 4:29 PM
MATH0500J	Feb 1, 2022 at 7:20 AM

Tangent Plane To Graph of $f : \mathcal{R}^n \to \mathcal{R}^1$ at point $(\mathbf{a}, f(\mathbf{a}))$

$$n = 2 : \; T(\mathbf{x}) = f(\mathbf{a}) + (f_x(\mathbf{a}), f_y(\mathbf{a})) \cdot (\mathbf{x} - \mathbf{a})$$

In general,

$$T(\mathbf{x}) = f(\mathbf{a}) + \nabla f(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$$

where $\nabla f(\mathbf{a}) = (f_1)(\mathbf{a}), f_2(\mathbf{a}, ..., f_n(\mathbf{a})$

Tangent Hyperplane

$n = 1$	Ordinary Tangent Line
$n = 2$	Tangent Plane

Example: $f(x,y,z) = \frac{x^2 y}{z}$
Note: $f : \mathcal{R}^3 \to \mathcal{R}^1$ so GRAPH lives in $\mathcal{R}^4$.
Find Equation of Tangent Hyperplane at $\mathbf{a} = (-3, 4, 2)$

$$f_x(x,y,z) = \frac{2xy}{z}$$

$$f_y(x,y,z) = \frac{x^2}{z} \ so \nabla f(x,y,z) = \left(\frac{2xy}{z}, \frac{x^2}{z}, -\frac{x^y}{z^2}\right)$$

$$f_z(x,y,z) = -\frac{x^y}{z^2}$$

at $\mathbf{a} = (-3, 4, 2)$: $f(\mathbf{a}) = \frac{(-3)^2 \times 4}{2} = 18$

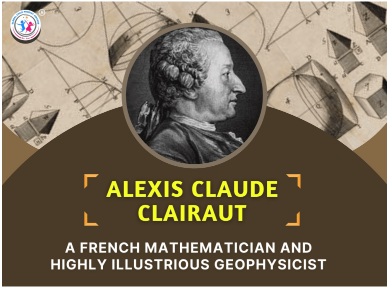
$$\nabla f(\mathbf{a}) = \left(\frac{(2)(-3)(4)}{2}, \frac{(-3)^2}{2}, \frac{-(-3)^2(4)}{2}\right) = \left(-12, \frac{9}{2}, -9\right)$$

Equation of Tangent Hyperplane is

$$w = 18 + \left(-12, \frac{9}{2}, -9\right) \cdot (x+3, y-4, z-2)$$

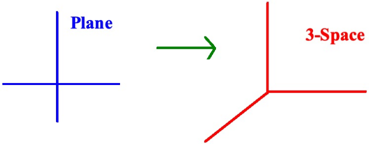
Clairaut's Theorem on Equality of Mixed Partials

If f_{xy} and f_{yx} are continuous at $\mathbf{a}$, then $f_{xy}(\mathbf{a}) = f_{yx}(\mathbf{a})$



May 7, 1713 – May 17, 1765

Parametrized Surfaces



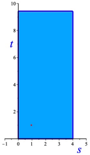
Function from $\mathcal{R}^2 \to \mathcal{R}^3$

Domain	Patch in Plane
Image	Surface in Space
Graph	Lives in $\mathcal{R}^5$

Need for Parametrizations: Graph of $f : \mathcal{R}^1 \to \mathcal{R}^1$ is a curve but not every curve is the graph of such a function

Similarly, graph of $f : \mathcal{R}^2 \to \mathcal{R}^1$ is a surface but not every surface is the graph of such a function.

Example: $\sigma(s,t) = (s\cos t, s\sin t, t), 0 \leq s \leq 4, 0 \leq t \leq 3\pi$

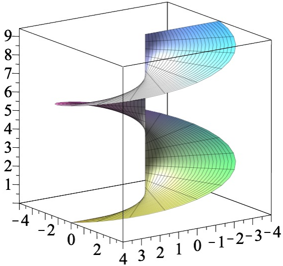


Point: $(1, \pi/4)$ so $\sigma(1, \pi/4) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

$\sigma_s(s,t) = (\cos t, \sin t, 0)$ and $\sigma_t(s,t) = (-s\sin t, s\cos t, 1)$

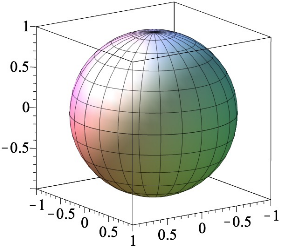
At $\left(1, \frac{\pi}{4}\right)$, representation of the tangent plane is

$$\sigma\left(1, \frac{\pi}{4}\right) + \sigma_s\left(1, \frac{\pi}{4}\right) s + \sigma_t\left(1, \frac{\pi}{4}\right) t$$



Parametrize Unit Sphere

$$\sigma(s,t) = (\cos t \cos s, \sin t \cos s, \sin s), 0 \le s \le 2\pi, 0 \le t \le 2\pi$$



$$x = \cos t \cos s, y = \sin t \cos s, z = \sin s$$

$$\begin{aligned} x^2 + y^2 + z^2 &= \cos^2 t \cos^2 s + \sin^2 t \cos^2 s + \sin^2 s \\ &= \cos^2 s(\cos^2 t + \sin^2 t) + \sin^2 s \\ &= \cos^2 s + \sin^2 s = 1 \end{aligned}$$

Parametrize Cylinder

$$x = s, y = 4\cos t, z = 4\sin t, 0 \le s \le 3, 0 \le t \le 2\pi$$