MATH 223

Some Hints and Answers for Assignment 25

Exercises 28abcd, 30, 31, 32 and 33 of Chapter 6.

28abcd: Hint: Review properties of the natural logarithm function.

(a)diverges.

(b) converges.

(c) diverges.

(d) diverges.

30: Switch to Polar Coordinates for (a). Treat (b) as optional extra credit problem.

(a) $\iint_{\mathbb{R}^2} \frac{1}{(1+x^2+y^2)^{3/2}}\, dA$
Solution

(b) $\iiint_{\mathbb{R}^3} \frac{1}{(1+x^2+y^2+z^2)^{3/2}}\, dV$
Solution

Solution:

31: Switch to polar coordinates. The integrand (a) Converges to π. (Recall $\int \cos^2\theta\, d\theta = \frac{\sin\theta\cos\theta+\theta}{2}$)

(b) Use polar coordinates again.Use integration by parts on $\int \ln r| : dr$. You will need to find $\lim_{a\to 0^+} a\ln a$ which is an indeterminate $0 \times -\infty$ form.. Use l'Hôpital's Rule with $a\ln a = \frac{\ln a}{1/a}$.
Final answer is-4π

32: Note that the unit disk D consisting of all points less than one unit from the origin lies entirely inside R so if the integral diverges on D, it will diverge on the larger set R as our function is always positive. Use polar coordinates

33: Let U be the set of points in $\mathbb{R}^3$ at least one unit from the origin; that is, $U = \{(x, y, z) : x^2+y^2+z^2 \geq 1\}$. Show that for $k > 5/2$, the triple integral

$$\mathcal{I} = \iiint_U \frac{(x^2+y^2)\ln(x^2+y^2+z^2)}{(x^2+y^2+z^2)^k}\, dV$$

converges and has value $\frac{16\pi}{3(2k-5)^2}$. Hint: Use spherical coordinates.

If $k > 5/2$, then $2k > 5$ so we can write $2k = 5 + p$ for some positive number p. Using spherical coordinates, the integral becomes

$$\iiint_U \frac{r^2\sin^2\phi\ln r^2}{(r^2)^k} r^2\sin\phi = \iiint_U \frac{r^4\sin^3\phi\ln r^2}{r^{2k}} = \iiint_U \frac{r^4\sin^3\phi(2\ln r)}{r^{5+p}} = \iiint_U 2\frac{\sin^3\phi\ln r}{r^{1+p}}$$