MATH 223

Hints and Answers for Assignment 24

Change of Variable

Exercise 21, 22, 24, 26 and 27 of Chapter 6

Exercise 21: Jacobian is $\begin{pmatrix} \cos v & -u\sin v \\ \sin v & u\cos v \end{pmatrix}$ with determinant u.

Exercise 22: Jacobian is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ v\,u^{v-1} & u^v\,\ln u \end{pmatrix}$.

Exercise 24: Begin with a sketch of the region $\mathcal{R}$. Show that in polar coordinates, this region is described as

$$\{(r,\theta) : 0 \le \theta \le \pi/4, \sec\theta \le r \le 2\sec\theta\}$$

and the function becomes $\sec\theta$. You may need to look up how to find an antiderivative of $\sec^3\theta$.

Exercise 26:. Show that the change of variable formula transforms I to

$$I = \int_{u=3}^{u=7} \int_{v=9}^{v=16} \frac{1}{2}\, dv\, du$$

which is half the area of the rectangle.

Exercise 27:
(a) Replace y with u/x in $v = y^2 - x^2$ and obtain a quadratic in x^2.

(b) This part is a bit messy but straightforward. One expression for the determinant is

$$\frac{2u}{\sqrt{4u^2+v^2}\sqrt{-2v+2\sqrt{4u^2+v^2}}\sqrt{2v+2\sqrt{4u^2+v^2}}}$$

. One chunk of the denominator has the form $\sqrt{-A+B}\sqrt{A+B} = \sqrt{B^2-A^2}$ where this last term reduces to $\sqrt{16u^2} = 4u$.

(d) Apply Change of Variable Theorem.