

MATH 223 Sample Exam 2

1. Suppose $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is the function given by $f(x, y) = (x^2y + 2y, xy + 1, \frac{\ln x}{y})$ and let $\mathbf{x}_0 = (1, 2)$.

(a) Compute the derivative matrix of f at \mathbf{x}_0 .

(b) Let $\mathbf{w} = (4, 8, 2019)$ and suppose g is a differentiable function from \mathbb{R}^3 to \mathbb{R}^2 such that $g(\mathbf{w}) = \mathbf{x}_0$ and $g'(\mathbf{w})$ is $\begin{pmatrix} -2 & 6 & 4 \\ 2 & -1 & 4 \end{pmatrix}$. Determine $(f \circ g)'(\mathbf{w})$.

2. The equations $\begin{cases} 3x^4y + y^3x^2 - 4t^3 = 4 \\ x + y + 2t - 6 = 0 \end{cases}$ implicitly define a curve $f(t) = (x(t), y(t))$ satisfying $f(2) = (-1, 3)$.

(a) Use implicit differentiation to determine dx/dt and dy/dt at $t = 2$

(b) Find an equation for the tangent line to f at $t = 2$.

3. NASA has given you the job of erecting a space probe on a newly discovered moon of Jupiter named Yrubelddim. To minimize interference with the probe's sensors, you must place the probe on the surface of the moon where its magnetic field is weakest. Yrubelddim is perfectly spherical with a radius of 3 thousand miles. Based on a coordinate system whose origin is at the center of Yrubelddim, the strength of the magnetic field in space is given by the function f whose value at the point (x, y, z) is $f(x, y, z) = xz - y^2 + 3x + 3$. Use the method of Lagrange multipliers to decide where on the surface of this moon you should locate the probe.

4. Otter Creek Brewing plans to market a new product, Warner Wonder Drink. Its advertising agency develops a campaign strategy built around the slogan "Warner's Works Wonders!" and estimates that if it spends $\$10,000x$ on TV/Radio advertising and $\$10,000y$ on newspaper and magazine ads, then the company will sell $1000z$ cases of Warner Wonder where $z = f(x, y) = 2x^3 + 18y^3 - 18x^2 - 108y^2 + 30x + 162y + 100$.

(a) Find all the critical points of f .

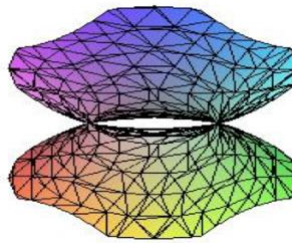
(b) Find $\frac{\partial^2 f}{\partial \mathbf{u}^2}$ where \mathbf{u} is an arbitrary unit vector.

(c) Use the Second Derivative Test to determine the nature of each critical point.

(d) Based on your analysis, how much would you recommend spending on TV/Radio advertising and how much on newspaper and internet ads if Otter Creek brewing desires to maximize its sales of Warner Wonder? Explain

5. (a) Pictured to the right is the surface whose equation in rectangular coordinates is $x^2 + y^2 - z^2 = 1$.

What is the equation of the surface in spherical coordinates? Simplify your answer as much as possible.



(b) An oil filter cartridge is a porous right-circular cylinder inside which oil diffuses from the axis to the outer curved surface. Describe the cartridge in *cylindrical coordinates* if the diameter of the filter is 4.5 inches, the height is 5.6 inches, and the center of the cartridge is drilled (all the way through) from the top to admit a 5/8-inch-diameter bolt.



6. Find the directional derivative at the point $(1, 0, 0)$ of the function $f(x, y, z) = x^2 + ye^z$ in the direction of the tangent vector at $g(0)$ to the curve in \mathbb{R}^3 defined parametrically by

$$g(t) = (3t^2 + t + 1, 2t, t^2)$$