Throughout this exam *E* denotes the ellipse in the (*xy*)-plane defined by the equation $\frac{x^2}{9}+\frac{y^2}{16}=1$ and *R* is the set of points on or inside the ellipse. The results of some calculations done on one part of a problem may be used on another part of a different problem without redoing the calculations.

1. Let *E* be the ellipse in the plane defined by the equation $\frac{x^2}{9}+\frac{y^2}{16}=1$ and let *R* be the set of points contained inside or on the ellipse.

(a) Sketch a picture of *E*.
(b) Show that *E* can be parametrized by $g(t)=(3\cos t, 4\sin t), 0\le t\le 2\pi$.
(c) Let $P\left(\frac{9}{5},\frac{16}{5}\right)$ be the point. Find an equation for the tangent line to the ellipse at this point *P*.
(d) Find a unit vector normal to the ellipse at this point *P*.
(e) Show that $|\, g'(t) \,|$ may be written as $\sqrt{9+7\cos^2 t}$
(f) Find the curvature of the ellipse at the point corresponding to $t=\pi/3$.
The following may be helpful:

$$U(t)=\frac{\cos t}{(9+7\,cos^2 t)^{\frac{1}{2}}} \text{ has } U'(t)=\frac{-9\,\sin t}{(9+7\,cos^2 t)^{\frac{3}{2}}},\quad V(t)=\frac{\sin t}{(9+7\,cos^2 t)^{\frac{1}{2}}} \text{ has } V'(t)=\frac{16\,\sin t}{(9+7\,cos^2 t)^{\frac{3}{2}}}$$

2. Again, Let *E* be the ellipse in the plane defined by the equation $\frac{x^2}{9}+\frac{y^2}{16}=1$ and let *R* be the set of points contained inside or on the ellipse.

a) Set up but do not evaluate an integral whose value would be the length of the ellipse.
b) Find the area of *R*.
c) Sketch a graph of the function $f(x,y)=\frac{x^2}{9}+\frac{y^2}{16}$ for $-3\le x\le 3$, $-4\le y\le 4$.
Describe the level curves for this function.
(d) Let **F** be the vector field $\mathbf{F}(x,y)=(x^2,xy)$. Find $\int_E \boldsymbol{F}$.
(e) Earlier in our course, we studied the temperature function $T(x,y)=2x^2+4y^2+2x+1$ defined on the unit disk. Find and classify all the critical points of *T* defined on the region *R* containing all the points inside or on the ellipse *E*. What are the warmest and coldest temperatures and where do they occur?

3. Let **F** be the vector field $\mathbf{F}(x,y,z)=\left(\cos x\, e^y z^3, \sin x\;\; e^y z^3, 3\sin x\, e^y z^2-\frac{1}{z}\right)$
(a) Find the divergence of **F** at the point $(\pi/2, 0, 1)$
(b) Find the curl of **F**.
(c) Show that **F** is a conservative vector field.
(d) Find a potential function for this vector field **F**.

4. Let *S* be the region between the unit circle *C* $(x^2+y^2=1)$ and our ellipse *E* ($\frac{x^2}{9}+\frac{y^2}{16}=1$).
(a) Is *S* simply connected? Explain.
(b) Let *Q* be the portion of the region *S* that lies in the first quadrant. Write the area of Q as a sum of double integrals in Cartesian coordinates. Do **not** evaluate the integrals.
Write the area of Q as a sum of double integrals in polar coordinates. Do not evaluate the integrals.
[Hint: Show the polar equation for the ellipse *E* has the form $r=\frac{12}{\sqrt{16\cos^2\theta+9sin^2\theta}}$ for points in *Q*.].
(c)

5. Let γ be the curve in 3-space parametrized by $\boldsymbol{g}(t)=(\sin t, \cos t, \sin t-\cos t), 0\le t\le 2\pi$.
Verify Stokes' Theorem for γ and the vector field $\mathbf{F}(x,y,z)=(\,yz, xz, xy\,)$