Multivariable Calculus: A Linear Algebra Approach
**Errata**
August 10, 2022

Preface: Page xiii, last line: " Duncan McCabe, an undergraduate mathematics major at Middlebury, " should be "Duncan McCabe and Cody Mattice, undergraduate mathematics majors at Middlebury, "

Chapter 1: Page 11: "We will usually write -(x)" should be "We will usually write - **x**"

Page 15, line 6: Delete **Invertibility**

Page 15, 3 lines from bottom replace "$+c_3$" with " $+c_3 - c_4$"

**Page 17: Add Section on Mathematical Induction**

Chapter 2: Page 28, Example 2: "8 f/sec" should be "8 ft/sec"

Page 31: "$(kg)^{-2}$" should be "$(\mathrm{kg})^{-2}$"

Page 34: In all of the equations replace lower case e with upper case E.

Page 41: Project 1: insert right parenthesis between $\beta t$ and $0 \leq t$

Chapter 3

Page 44, line 2: "An individual" should be "If an individual"

Page 47, Example 2: $fx$ should be $f_x$, $fy$ should be $f_y$ and $fz$ should be $f_z$.

Page 48, **Alternative Notation** replace "notation $\frac{\partial f}{\partial x}$" with "notation $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$"

Page 57: Second **Example 2** should be **Example 3**

Page 58: **Example 3** should be **Example 4**. Also " $D " should be " $$D$ "

Page 58, 6 lines from bottom: Add colon (:) after **Solution**

Page 60, Exercise 20: " let " should be " Let "

Page 62 , Exercise 7: Line 1: $(x_4, x_5, 6)$ should be $(x_4, x_5, x_6)$.
Line 2: Replace existing text with. following:
(a) Determine $f(10, 23, 18, 12, 30, 21)$.
(b) Describe the set of points $(x, y, z)$ in space such that $f(1, 1, 1, x, y, z) = 0$.

Chapter 4: Page 70: Line 6 of **Proof of (d):** $\mathcal{M}_2, \mathcal{M}_2$ should be $\mathcal{M}_1, \mathcal{M}_2$

Page 71, Example 2:: $\cos xe^y$ should be $\cos x + e^y$

Page 81, 3rd line of Example 3: Change "length of $\mathbf{v} =$" to "length of $\mathbf{v}$. which is "

Page 84, Line 2: delete period in "joining them as."

Page 86, Second line of Section 4.6: $-1 \leq \cos\theta \leq$ should be $-1 \leq \cos\theta \leq 1$

Page 89, Second Line of 4.6.2 Newton's Method: $x - 2 = 0$ should be $x^2 - 2 = 0$

Page 96, Exercise 17: Delete left parenthesis ( from "for $(x \neq 0$

Page 98, Line 2: right after "vector - valued functions", add "in Exercises 30 - 32"

Page 99, Exercise 39: Change "1,00,000" to "1,000,000"

Chapter 5 **: Need to Number the Theorems.**

Page 101: First Line of 5.1.1: Change " **Theorem: The Little Chain Rule** " to " **Theorem 5.1.1 (*The Little Chain Rule* ):**"

Page 103: First Line of 5.1.2: Change " **Theorem: General Chain Rule** " to " **Theorem 5.1.2 (*General Chain Rule* ):**"

Page 104, Line 1: Insert colon (:) immediately after **Solution**

Page 107, second line after "so": $= 9f({}^*_u)^2 + 12f_u^*f_v^* + 4(f_v^*)^2 + 4f({}^*_u)^2 - 12f_u^*f_v^* + 9f({}^*_v)^2$
should be $= 9(f_u^*)^2 + 12f_u^*f_v^* + 4(f_v^*)^2 + 4(f_u^*)^2 - 12f_u^*f_v^* + 9(f_v^*)^2$

Page 109, Line 7 in Section 5.3: " $\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2}$ " should be " " $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}$ "

Page 110: 4th line in Example 3: "The text " should be "The test "

Page 113: In paragraph "Let's look a bit": Change "differentiable function real-valued" to "differentiable real-valued"

Page 121, Example 11: Both instances of " \$C " should be " \$$C$ "

Page 121, 2 line from bottom: " $3xx$ " should be " $3x$ "

Page 124, First Definition: "*Hessian Matrix* " should be "**Hessian Matrix**"

Page 124, Second Definition: "*Positive-Definite Matrix* " should be "**Positive-Definite Matrix**" and " *positive semi-definite*" should be " **positive semi-definite**"

Page 129, in Example 2, change "You can" to "We can "

Page 131, Sentence beginning "Figure 5/17 illustrates ": insert period at end of sentence.

Page 139, Exercise 37. Delete period (.) in first line.

Page 139, Exercise 42, last line: Add right parenthesis so $(\mathbf{c}, f(\mathbf{c})$ becomes $(\mathbf{c}, f(\mathbf{c}))$

Page 139, Exercise 48: add:
(a): Show that $f(x, y) = 4x^2 + 8xy + 5y^2$ is a positive-definite function.
(b) Show that $g(x, y) = -x^2 + 2xy - 3y^2$ is a negative-definite function

Page 141, Exercise 63: Change " \$T " to " \$$T$ "

Page 142, Exercise 64: Change " \$p " to " \$$p$", " \$q " to " \$$q$ ", and " \$B " to " \$$B$"

Page 142, Exercise 65: Change " \$C " to " \$$C$ "

Chapter 6

Page 158: italicize $x$ in **x-simple** and the $y$ in **y-simple**

Page 160, 7 lines from bottom: delet period in "such that."

Page 161, 8 lines from bottom: " with mess less than " should be "with mesh less than "

Page 162, **Proof of (2):** Change "use" to "Use"

Page 163, Line 7: Insert line break just before **Partition Theorem**

Page 167, Line 3; Insert equality sign (=) between $\frac{1}{2}(u-1)^{-1}$ and $\frac{1}{2\sqrt{u-1}}$.

Page 174, Line 3 of **Solution**: Add colon after *Method A*

Page 175, 8 lines from bottom: "as $as \to 0$ " should be " as $a \to 0$ "

Page 175, 3 lines from bottom "$\int_D f\, DA$" should be " $\int_D f\, dA$ "

Page 182, Exercise 8: " $x - axis$ " should be "$x$-axis "

Page 185, Exercise 32: Add at the start: "Determine the convergence or divergence of the integral"

Page 185,k Exercise 35b: Formula at end of the line should read

$$\frac{1}{\lambda}\left[\lambda^2 - 1 + e^{-3\lambda}(11 + 3\lambda)\right]$$

Chapter 7:

Page 191, About 5 lines from bottom: Equation with $Work =$: "Work" should not be italicized

Page 196, 9 lines from bottom: " $0 \le s11$ " should be " $0 \le s \le 1$"

Page 201, 4th line in Solution of Example 6: " $\cos^t$ " should be " $\cos^2 t$ "

Page 201, 8th line in Solution of Example 6: $\mathcal{L}(\gamma) = \int_0^{\pi/2} \int |\mathbf{g}'(t)|\, dt =$ should be $\mathcal{L}(\gamma) = \int_0^{\pi/2} |\mathbf{g}'(t)|\, dt =$

Page 201, 11th line in Solution of Example 6: $\mathcal{L}(\gamma) = \int_0^{\pi/2} \int |\mathbf{g}'(t)| dt =$ should be $\mathcal{L}(\gamma) = \int_0^{\pi/2} |\mathbf{g}'(t)| dt =$

Page 202, Example 7:" $4\pi)$ " should be " $4\pi$ "

Page 205, 7 lines from bottom: "straight forward" should be "straightforward"

Page 211, 2nd line of proof of Theorem 6.2: " $\mathbf{T}' = \frac{d\mathbf{T}}{t}$ " should be " $\mathbf{T}' = \frac{d\mathbf{T}}{dt}$ "

Page 220, Exercise 12: " $2xx^2$ " should be " $2x^2$ "

Page 220, Exercise 14: " $\mathbf{g}$ " should be " $\mathbf{g}(t)$ "

Page 220, Exercise 15: " $\mathbf{h}s$ " should be " $\mathbf{h}(t)$ "

Page 222, Exercise 26: " cosh " should be " sinh "

Page 222, Exercise 28: " **y-axis** " should be " $y$-axis "

Page 222, Exercise 30: Sketch the solid obtained by revolving the graph of $y = 4\sqrt[3]{x}$ from (8,8) to (27, 12) around the $y-$axis and determine its surface area.

Page 223, Replace Exercise 41 of Chapter. 7 with:

For each of the following, sketch the curve with the given equation and determine its curvature:

(a) $y = x^2$

(b) $y = x^3$

(c) $y = x^4$

(d) $y = x^5$

(e) $y = x^{-1}, x > 0$

(f) $y = x^{-2}, x > 0$

(g) $y = \ln x, x > 0$

Page 224, Exercise 48(b): " $-\frac{2A}{5}$ " should be " $\frac{2A}{5}$ "

Page 224, Exercise 49: " $7x$ " should be " $-7x$ "

Page 224, Exercise 50: " $-x + 3y$ " should be " $4x + y$ "

Page 225, Exercise 55(e): " $v_1 v_2 V_2$ " should be $v_1 v_2 v_3$ "

Page 226, **Art Gallery Problem**: " $\lfloor 3 \rfloor = \lfloor 4 \rfloor = \lfloor 5 \rfloor = 1$, but $\lfloor 6 \rfloor = 2$. " should be " $\lfloor 3/3 \rfloor = \lfloor 4/3 \rfloor = \lfloor 5/3 \rfloor = 1$, but $\lfloor 6/3 \rfloor = 2$. "

Page 230, Add colon after **Example 3**

Chapter 8:

Page 229, 2nd line from bottom: "For points above this line, div $\mathbf{F}$ is positive " should be "For points below this line, div $\mathbf{F}$ is positive "

Page 248: Line. 1: " $X2$ " should be " $x_2$ "

Page 258, 9 lines from bottom: " $dxdydx$ " should be " $dxdydz$ "

Page 276. Exercise 21. (a) and (b): "$- \leq t$ " should be " $0 \leq t$ "

Page 275, Exercise 5: " $m \times n$ " should be " $n \times n$ "

Page 276, Exercise 18: " $(y + 2x, x + 3x, 2x + 3y)$ " should be " $(y + 2x, x + 3z, 2z + 3y)$ "

Page 276, Exercise 21(a) and (b) " $- \leq t$ " should be " $0 \leq t$"

Page 277, Exercise 29(d), 3rd line: " $g(\sigma(S, t))$ " should be " $f(\sigma(s, t))$ "

Page 277, Exercise 31: The vector field should be $\mathbf{F}(x, y, z) = (x, 2y, 3z)$.

Chapter 9:

Page 290, Line 2: " $partial R$ " should be " $\partial R$ "

Page 290, 2nd line of Proof of Theorem: The dash (-) should be an equal sign (=)

Page 291: Replace Exercises 1 - 5 with:

**1.** Find $\int_\gamma \omega$ if $\omega = \frac{-y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy$ and $\gamma$ is the circle of radius 2, centered at the origin and traced out counterclockwise.

**2.** Find $\int_\gamma \omega$ if $\omega = x\,dx + dy$ and $\gamma$ the graph of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ traced out counterclockwise.

**3.** Find $\int_\gamma \omega$ if $\omega = y\,dx + x\,dy$ and $\gamma$ has the parametrization $g(t) = (\cos t, \sin t, t), 0 \le t \le 2\pi$.

**4.** Find $\int_\gamma \omega$ if $\omega = (x+y)\,dx + zy^2\,dy + (x-z)dz$ and $\gamma$ in the line segment from (1, 2, 3) to (4, $-3$, 7).

**5.** Find $\int_\gamma \omega$ if $\omega = (x^2+yz)\,dx + z\,dy + (y-x)dz$ and $\gamma$ has parametrization $g(t) = (2t+3, 3t-2, t+4), 0 \le t \le 2$.

Page 291, Exercises 3, 4, 5: " $gamma$ " should be " $\gamma$ "

**IF POSSIBLE, ADD THE FOLLOWING AT END OF SECTION 1.2.4 BEFORE SECTION 1.3**:

**Example:** Suppose $\lambda \neq \mu$ are distinct eigenvalues of the matrix $A$ with corresponding eigenvectors $\mathbf{v}$ and $\mathbf{u}$. Thus $\mathbf{v}$ and $\mathbf{u}$ are nonzero vectors with $A\mathbf{v} = \lambda\mathbf{v}$ and $A\mathbf{u} = \mu\mathbf{v}$. Show that $\{\mathbf{v}, \mathbf{u}\}$ is a linearly independent set.

*Solution:* Suppose $c$ and $d$ are constants such that

$$(*)\;\; c\mathbf{v} + d\mathbf{u} = \mathbf{0}.$$

We need to show that both $c$ and $d$ must be 0. Let's multiply both sides of (*) by the matrix $A$:

$$A(c\mathbf{v} + d\mathbf{u}) = A(c\mathbf{v}) + A(d\mathbf{u}) = cA\mathbf{v} + dA\mathbf{u} = c\lambda\mathbf{v} + d\mu\mathbf{u} = A\mathbf{0} = \mathbf{0}.$$

Thus we have

$$(**)\;\; c\lambda\mathbf{v} + d\mu\mathbf{u} = \mathbf{0}.$$

If we multiply (*) by $\lambda$, we obtain

$$(***)\;\; c\lambda\mathbf{v} + d\lambda\mathbf{u} = \mathbf{0}.$$

Subtract equation (***) from (**) to obtain

$$d(\mu - \lambda)\mathbf{u} = \mathbf{0},$$

but $\mu - \lambda$ is a nonzero number since the eigenvalues are distinct. Hence $d\mathbf{u} = \mathbf{0}$ which implies $d = 0$ because $\mathbf{u}$ is a nonzero vector. Equation (*) reduces to $c\mathbf{v} = \mathbf{0}$ which also implies $c = 0$ by the nonzero character of the vector $\mathbf{v}$.

**Add a New Exercise 24 for Chapter 1**:

**Exercise:** Let $\lambda_1, \lambda_2, \lambda_3$ be three distinct eigenvalues of the same matrix $A$ with corresponding eigenvectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$. Prove that the set of these eigenvectors is linearly independent.