MATH 223: Multivariable Calculus

Vector Differential Calculus

Curvilinear Coordinates

Polar Coordinates (r, θ)

For any point **x** = (*x*,*y*) in the plane, there are numbers r and θ, called the ***polar coordinates*** of **x** such that

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Spherical Coordinates (r, ϕ, θ)

For any point **x** = (*x*,*y*,*z*) in three dimensional space, there are numbers r and θ and ϕ, called the ***spherical coordinates*** of **x** such that

$$x = r\sin\phi\cos\theta$$
$$y = r\sin\phi\sin\theta$$
$$z = r\cos\phi$$

Cylindrical Coordinates(r, θ, z)

For any point **x** = (*x*,*y*,*z*) in three dimensional space , there are numbers *r*, θ and *z* called the ***cylindrical coordinates*** of **x** such that

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

Jacobian Matrices

$$\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \quad \begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad . \begin{pmatrix} \sin\phi\cos\theta & r\cos\phi\cos\theta & -r\sin\phi\sin\theta \\ \sin\phi\sin\theta & r\cos\phi\sin\theta & r\sin\phi\cos\theta \\ \cos\phi & -r\sin\phi & 0 \end{pmatrix}$$