MATH 223 Fall 2022
Assignment 6
Due: Monday, September 26

Reading

Read carefully Sections 3.4 "Parametrized Surfaces" in our text *Multivariable Calculus: A Linear Algebra Based Approach.*

Writing

Write out careful and complete solutions of Exercise 12 in Chapter 3 and the exercises below. (If you have the earlier version of the text, this is Exercise 10)

1. In class, we examined the function given by $f(x, y) = \frac{xy}{x^2 + 2y^2}$.

      (a) Show that 3/19 is a possible value for this function by exhibiting a specific point (a,b) such that $f(a, b) = \frac{3}{19}$.

      (b) Show that there is no point (a,b) such that $f(a, b) = 1$.

      (c) What is the largest possible value **M** of this function?

[*Hint* for this problem: every point in the plane not on the vertical axis lies on a line with equation $y = mx$ for some constant m.]

2. Let g be the function defined by $g(x, y) = \frac{xy}{2x^2 + 3y^2}$.

    (a) What is the domain of this function?

    (b) Show that $\lim_{(x,y) \to (0,0)} g(x, y)$ does not exist.

    (c) For which points (x,y) in the plane is $g(x,y) > 0$? For which points if $g(x,y) < 0$?

    (d) What is the image of this function? You should carefully describe the set of real numbers which are possible values for this function.

3. Let f be the function defined by $g(x, y) = \frac{x^2 y}{x^4 + y^2}$.

  (a) Show that the limit of f as (x,y) approaches the origin along any line is 0.

  (b) Show that the limit of f as (x,y) approaches the origin along the curve $y = x^2$ is ½.

  (c) Does $\lim_{(x,y) \to (0,0)} f(x, y)$ exist? Justify your answer.