MATH 223 Fall 2022
Assignment 16
Due: Monday, October 24

Reading

Read carefully Section 5.5 "Implicit Differentiation" in our text *Multivariable Calculus: A Linear Algebra Based Approach.*

Writing

Write out careful and complete solutions of Exercises 1, 2, and 3 below.

1. Let $f(x,y) = 2x^2 + 3y^2$ be a real-valued function defined on the plane.
   (a) Let **P** be the point (5,4) and *C* the level curve of *f* containing **P**. Identify the nature of *C:* Is it a Circle? Parabola? Pair of Lines? Sketch a picture of *C.*
   (b) Use classic implicit differentiation or some other method to find an equation for the line *L* tangent to *C* at **P**.
   (c) Determine the gradient vector **v** of f at **P**.
   (d) Show that this gradient vector is orthogonal to any vector lying along *L*.

2. Let $g(x,y) = 2x^2 - 3y^2$ be a real-valued function defined on the plane.
   (a) Let **P** be the point (5,4) and *C* the level curve of g containing P. Identify the nature of *C:* Is it a Circle? Parabola? Pair of Lines? Sketch a picture of *C.*
   (b) Use classic implicit differentiation or some other method to find an equation for the line *L* tangent to *C* at **P**.
   (c) Determine the gradient vector **v** of g at **P**.
   (d) Show that this gradient vector is orthogonal to any vector lying along *L*.

3. (Williamson and Trotter) A spaceship traveling in the plane along a path such that at time $t \geq 0$, the ship is at position $g(t) = (3t^2, t^3)$. The intensity of gamma radiation at the point (*x*,*y*) in the plane is $I(x,y) = x^2 - y^2$,
   wherever $I(x,y) \geq 0$. Describe fully, using a labeled graph where appropriate, the following:
   (a) The level curve of *I* the ship is on at $t = 1$.
   (b) The path of the ship for $t \geq 0$.
   (c) The gradient vector of *I* at the ship's position when $t = 1$.
   (d) The ship's velocity vector at $t = 1$.
   (e) The time – if there is one – when the ship stops increasing its radiation risk and begins its
   race to safety. Does its course become more dangerous later on?