Reading

Read carefully Sections 4.3 "Directional Derivatives" in our text *Multivariable Calculus: A Linear Algebra Based Approach.*

Writing

Write out careful and complete solutions of Exercises 17 and 18 in Chapter 4 as well as Problems A, B, and C below.

Problem A: For each of these functions f find gradient $\nabla f(\mathbf{x})$ of f at a general point in the domain of f:
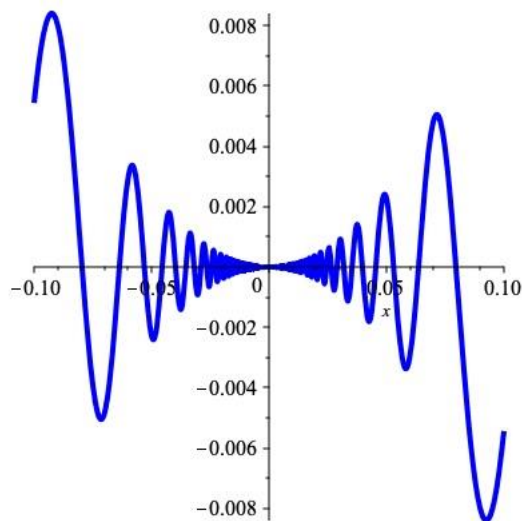(1) $f(x, y) = 2x^3 - 3y^2$
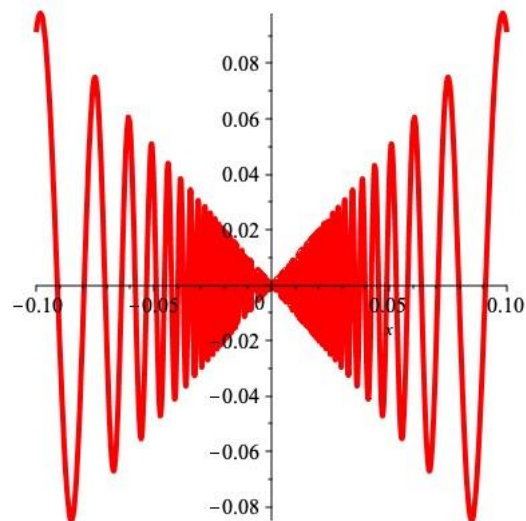(2) $f(x, y, z) = (5x - 7y)z$
(3) $f(x_1, x_2, x_3) = \frac{x_1 x_3}{x_2}$

Problem B: Write an equation in terms of the coordinate variables (x, y, z) for the tangent hyperplane for $f(x, y, z) = 2x^2 - y^2 + 3z^2$ when $x = y = z = 1$.

Problem C: Let f be the real-valued function $f:\ \mathbb{R}^p \to \mathbb{R}$ defined by $f(\mathbf{x}) = |\mathbf{x}|^2 = \mathbf{x} \bullet \mathbf{x}$. If $p = 2$, prove that $\nabla f(\mathbf{x}) = 2\mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^p$. Is this result true for other values of p?

Graph of
$$f(x) = x^2 sin\left(\frac{1}{x}\right)$$

Graph of
$$f(x) = 2x\, sin\left(\frac{1}{x}\right)$$