2D geometry basics: in principle

1. Points and vectors in the Euclidean plane:
   (a) What do we mean by ***points*** and ***vectors*** in the plane?
   In what ways are these concepts similar? In what ways are they distinct?
   (b) How do we conventionally denote points and vectors in writing?
   (c) Discuss the notion of the ***weight*** of points and vectors.
   (d) What is the fundamental operation that can be performed on vectors, and how is this restricted when we apply it to points?
   (e) What are the key relationships between points and vectors?

2. Suppose that $\vec{v}$ and $\vec{w}$ are vectors in the plane.
   (a) Give the geometric meaning of the ***dot product*** $\vec{v} \cdot \vec{w}$; what does it allow us to compute about a vector or pair of vectors?
   (b) Give the geometric meaning of the ***scalar cross-product*** $(\vec{v} \times \vec{w}) \cdot \hat{k}$; what does this allow us to compute?

3. Cartesian coordinates:
   (a) Describe the Cartesian plane, including the meaning of its ***origin*** and its x- and y-***axes***.
   What is *unnatural* about imposing such coordinates on the plane? Why is it nonetheless useful?
   (b) How do we represent *points* and *vectors* using Cartesian coordinates?
   What should we be extremely careful about when doing so?
   (c) How can our basic operations of [affine/linear] combinations be performed in Cartesian coordinates?
   (d) How can we compute the *dot product* and *scalar cross-product* of two vectors in Cartesian coordinates?

4. How can we use ***affine coordinates*** to track "weights" of combinations of points and vectors?

. . . and in practice

5. Find the "weights" of the following combinations of points and vectors, and classify each as a *vector*, a *point*, or *neither*:
   (a) $p + q + r$ (b) $\frac{1}{3}p + \frac{1}{3}q + \frac{1}{3}r$ (c) $p + \vec{v}$ (d) $q + 3\vec{v} - \vec{w}$
   (e) $\vec{r} - 5\vec{v} + 4\vec{w}$ (f) $p + q - r$ (g) $p - \frac{1}{2}q$

6. Find the lengths of each of the following vectors, and for each pair of them, determine whether they form an *acute*, *obtuse*, or *right* angle:
   (a) $\langle 2, -1\rangle$ (b) $\langle 3, 4\rangle$ (c) $\langle 1, 2\rangle$ (d) $\langle -3, 0\rangle$

7. For each of the pairs of vectors below, find the area of the triangle they span, and determine whether the smaller angle from the first vector to the second goes *counterclockwise* or *clockwise*:
   (a) $\langle 1, 3\rangle$ and $\langle 3, 1\rangle$ (b) $\langle 2, 0\rangle$ and $\langle -1, 1\rangle$

8. Suppose that $p = (2, 1)$, $q = (0, 3)$, $r = (0, 0)$, $\vec{v} = \langle -1, -1\rangle$, $\vec{w} = \langle 4, 0\rangle$, and $\vec{r} = \langle 0, 0\rangle$.
   Write each of these in *affine coordinates*, and use this to compute the affine coordinates for each of the combinations from problem 5.