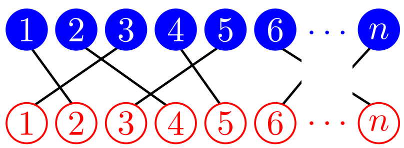
Permuting n distinct elements

Arrange n elements into a list, *without replacement.*

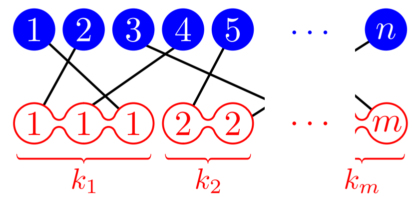


(bijection)

$$n! = n(n-1)(n-2)\cdots 2\cdot 1$$

Multinomials: $n = k_1 + k_2 + \cdots + k_m$, all $k_i \geq 0$

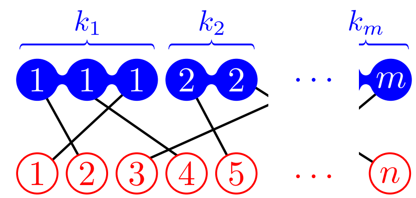
Form m labeled groups of given sizes from n distinct elements.



(bijection)

$$\binom{n}{k_1\ k_2\ \cdots\ k_m} = \frac{n!}{k_1!\,k_2!\,\cdots\,k_m!}$$

Arrange the elements of m labeled groups of given sizes into a list of length n.

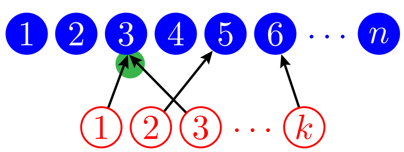


(bijection)

$$\binom{n}{k_1\ k_2\ \cdots\ k_m} = \frac{n!}{k_1!\,k_2!\,\cdots\,k_m!}$$

Choosing k elements from n distinct elements

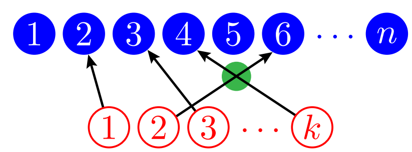
Ordered list of k elements, *with replacement.*



(function $K \to N$)

$$n^k$$

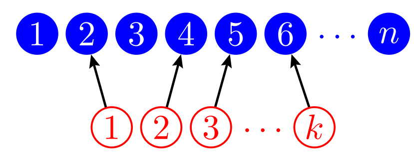
Ordered list of k distinct elements, *without replacement.*



(injection $K \to N$)

$$P(n,k) = \frac{n!}{(n-k)!} = n(n-1)\cdots(n-k+1)$$

Unordered group of k distinct elements, *without replacement.*

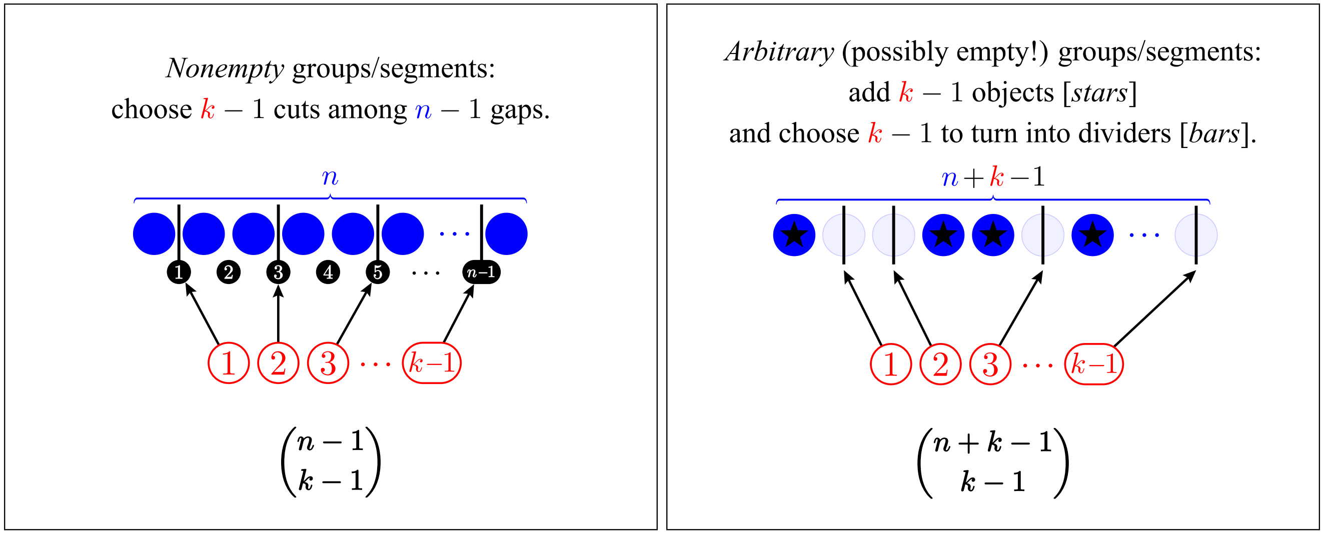


(increasing function $K \to N$)

$$\binom{n}{k} = C(n,k) = \frac{n!}{k!\,(n-k)!}$$

permute K
(factor of $k!$)

Grouping *n* identical elements into *k* labeled groups
(or splitting a list of length *n* into *k* labeled segments)



Grouping *n* distinct elements into *k* groups